

§5.4 Graphs of tangent, cot, sec(x), cosec(x).

(28)

$\tan(x)$, $\cot(x)$ period π

$\csc(x)$, $\sec(x)$ period (2π) .

Transformations

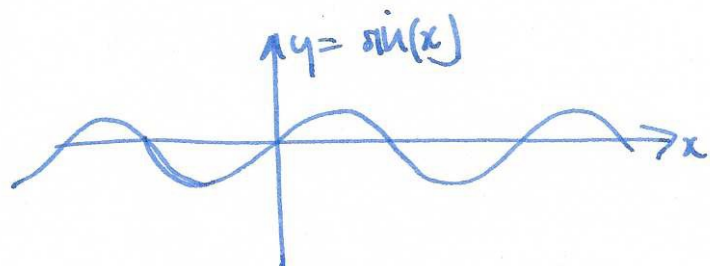
$$\left. \begin{array}{l} y = a \tan(kx) \\ y = a \cot(kx) \end{array} \right\} \text{ period } \frac{\pi}{k}$$

Examples graph $y = \tan(2x)$ $y = \tan 2(x - \frac{\pi}{4})$.

$$\left. \begin{array}{l} y = a \csc(kx) \\ y = a \sec(kx) \end{array} \right\} \text{ period } \frac{2\pi}{k}$$

Examples $y = \frac{1}{2} \csc(2x)$, $y = \frac{1}{2} \csc(2x + \frac{\pi}{2})$.

§5.5 Inverse trig functions



Now
one-to-one so there is an inverse
we call $\sin^{-1}(x)$ or $\arcsin(x)$.

inverse function property:

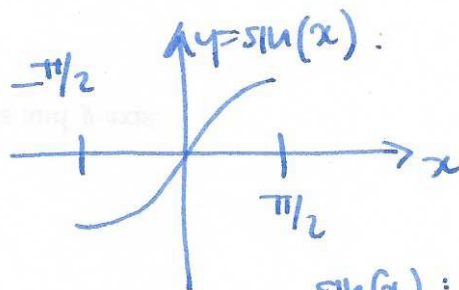
$$y = \sin(x) \Leftrightarrow \sin^{-1}(y) = x$$

$$\sin(\sin^{-1}(x)) = x \quad \text{for } -1 \leq x \leq 1$$

$$\sin^{-1}(\sin(x)) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

not one-to-one!
no inverse!

restrict domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$.



$$\sin(x) : [-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow [-1, 1]$$

$$\sin^{-1}(x) : [-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$$

Examples find $\sin^{-1}(\frac{1}{2})$ $\sin^{-1}(-\frac{1}{2})$

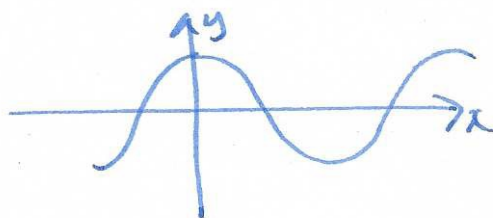
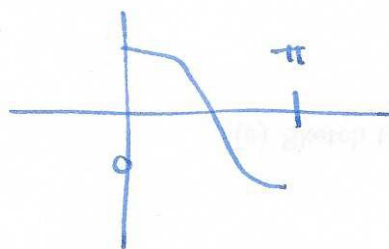
calculator: $\sin^{-1}(0.32)$

find $\sin^{-1}(\sin(\frac{\pi}{3})) = \frac{\pi}{3}$.

Warning: $\sin^{-1}(\sin(\frac{2\pi}{3})) \neq \frac{2\pi}{3}$
 $= \frac{\pi}{3}!$

Inverse $\cos(x)$

restrict domain



not one-to-one.
no inverse.

$$\cos(x) : [0, \pi] \rightarrow [-1, 1]$$

$$\cos^{-1}(x) : [-1, 1] \rightarrow [0, \pi]$$

$$y = \cos(x) \Leftrightarrow \cos^{-1}(x) = y$$

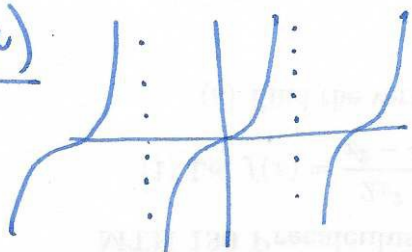
$$\cos^{-1}(\cos(x)) = x \quad \text{for } -1 \leq x \leq 1$$

$$\cos(\cos^{-1}(x)) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

Examples find: $\cos^{-1}(\frac{\sqrt{3}}{2})$ $\cos^{-1}(0)$ $\cos^{-1}(\frac{5}{7})$

find $\cos^{-1}(\cos(\frac{2\pi}{3}))$ $\cos^{-1}(\cos(\frac{5\pi}{3}))$.

$\tan^{-1}(x)$



restrict domain to $(-\frac{\pi}{2}, \frac{\pi}{2})$.

$$\tan(x) : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow (-\infty, \infty)$$

$$\tan^{-1}(x) : (-\infty, \infty) \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$$