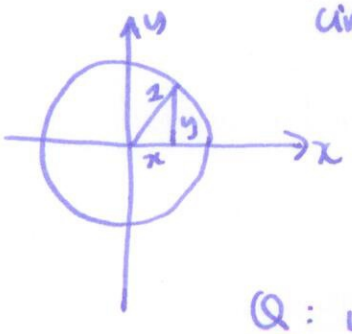
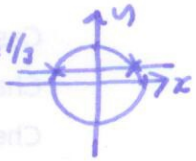
§5.1 The unit circle



circle of radius 1 $x^2+y^2=1$.

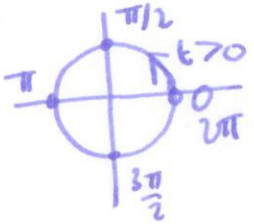
Quadrants. II | I / III | IV

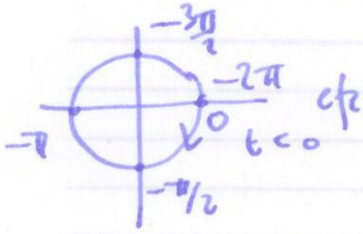
Q: which points on the circle have y-coord $\frac{1}{3}$? 

$x^2+\left(\frac{1}{3}\right)^2=1 \qquad x^2=\frac{8}{9} \qquad x=\pm\frac{2\sqrt{2}}{3}$.

Q: which point in quadrant II has $y=\frac{1}{3}$? $\left(-\frac{2\sqrt{2}}{3},\frac{1}{3}\right)$.

Distance round the circle: circumference is $2\pi r$ $r=1$ 2π.

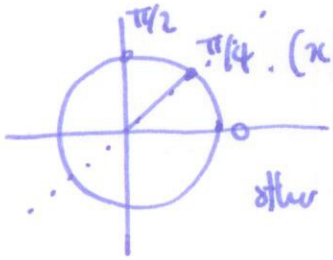




Q: find terminal point corresponding to 3π, $-\frac{5\pi}{2}$.

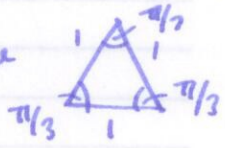
Special points

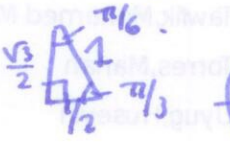
t	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
terminal point	$(1,0)$	$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$	$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$	$(0,1)$

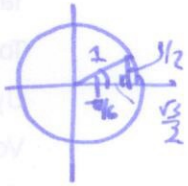


(x,y) with $x=y$ so $x^2+y^2=1$ $2x^2=1$

$x^2=\frac{1}{2}$ $x=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$.

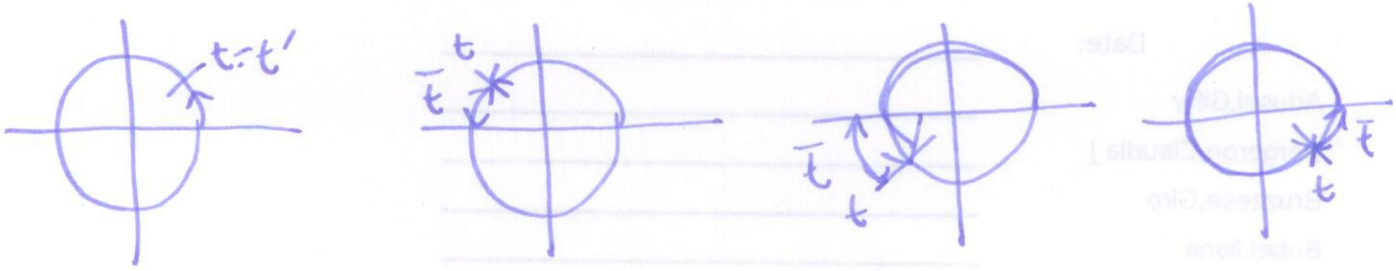
other values from special triangle 



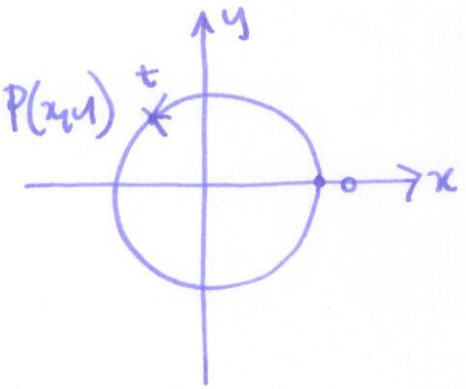


Q: find terminal point for $\frac{3\pi}{4}$, $-\frac{5\pi}{6}$.

Reference number: $\bar{t}$ shortest distance to x-axis.



§5.2 Trigonometric functions of real numbers



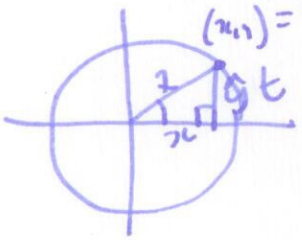
Defn Let $P(x,y)$ be the terminal point corresponding to t. Then

$$\left.\begin{aligned}\sin(t) &= y\\ \cos(t) &= x\end{aligned}\right\} \qquad \tan(t) = \frac{\sin(t)}{\cos(t)} = \frac{y}{x}$$

$$\csc(t) = \frac{1}{\sin(t)} = \frac{1}{y} \qquad \cot(t) = \frac{1}{\tan(t)} = \frac{x}{y}.$$

$$\sec(t) = \frac{1}{\cos(t)} = \frac{1}{x}$$

compare with right angled triangles:



$$\sin(t) = \frac{\text{opp}}{\text{hyp}} \qquad \tan(t) = \frac{\text{opp}}{\text{adj}}.$$

$$\cos(t) = \frac{\text{adj}}{\text{hyp}}$$

angle measured in t is 1 radians.

Find $\cos\left(\frac{2\pi}{3}\right)$, $\tan\left(-\frac{\pi}{3}\right)$, $\sin\left(\frac{19\pi}{4}\right)$.