

§4.5 Exponential and logarithmic equations

recall exponent rules:

- $a^x a^y = a^{x+y}$
- $(a^x)^y = a^{xy}$
- $a^0 = 1$
- $a^{-x} = \frac{1}{a^x}$

Solve: ① $2e^{3-4x} = 7$

$$e^{3-4x} = \frac{7}{2}$$

$$\ln(e^{3-4x}) = \ln(\frac{7}{2})$$

$$3-4x = \ln(\frac{7}{2})$$

$$3-\ln(\frac{7}{2}) = 4x$$

$$x = \frac{3-\ln(\frac{7}{2})}{4}$$

② $\log_2(x+3) = 12$

$$2^{\log_2(x+3)} = 2^{12}$$

$$x+3 = 2^{12}$$

$$x = 2^{12} - 3.$$

③ $\log(x+2) + \log(x-1) = 4$

$$e^{\log((x+2)(x-1))} = e^4$$

$$(x+2)(x-1) = e^4$$

$$x^2 + x - 2 - e^4 = 0$$

(quadratic formula)

log rules:

- $\log_a(xy) = \log_a(x) + \log_a(y)$
- $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
- $\log_a(x^y) = y \log_a(x)$
- $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

④ (quadratic type).

$$e^{2x} - e^x = 46$$

$$(e^x)^2 - e^x - 46 = 0$$

$$(e^x - 3)(e^x + 2) = 0$$

$$e^x = 3 \quad e^x = -2$$

$$x = \ln(3) \quad \text{no solns.}$$

⑤ factor of e^x :

$$3x e^x + x^2 e^x = 0$$

$$e^x (3x + x^2) = 0$$

$$e^x x (3+x) = 0$$

$$x = 0, -3.$$