§3.2 Polynomial functions and their graphs

Polynomial of degree n: $\quad a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

($a_n \neq 0$) $\quad$ numbers $a_0, a_1, \ldots, a_n$ are the <u>coefficients</u>

a_0 is the <u>constant coefficient</u>.

a_n is the <u>leading coefficient</u> $\quad a_n x^n$ is the leading term.

<u>Example</u>

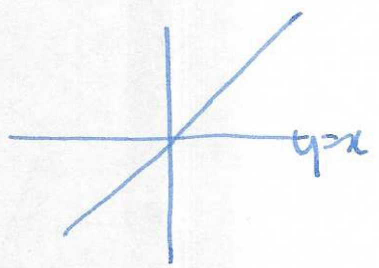
$$\underbrace{2}_{\text{leading coefficient.}}x^4 + x^2 - 3x + 2 \qquad \text{degree } 4$$

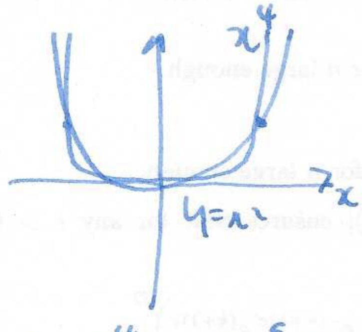
coefficients:

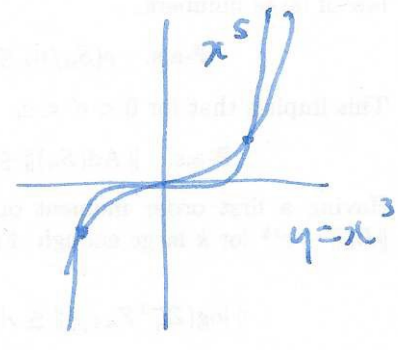
x^4	x^3	x^2	x^1	x^0
2	0	1	-3	2.

Graphs of polynomials are continuous (no breaks, holes or jumps or corners).

<u>Examples</u>







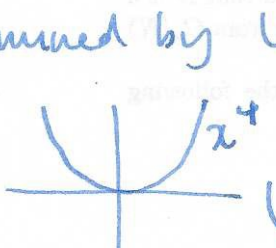
<u>Transformations</u>: Examples: $-x^3$, $(x-2)^4$, $-2x^5+4$.

<u>End behaviour</u>: $\quad x \to \infty$ means "x gets large in the +ve direction"

$x \to -\infty$

<u>Fact</u>: end behaviour determined by leading term.

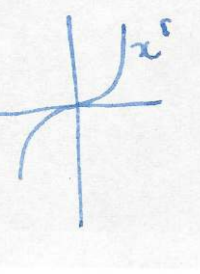
<u>Example</u> $p(x) = 2x^4 - 3x^3 - 10$.

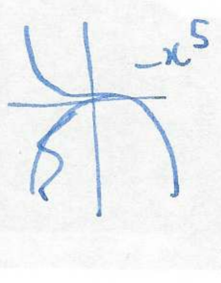


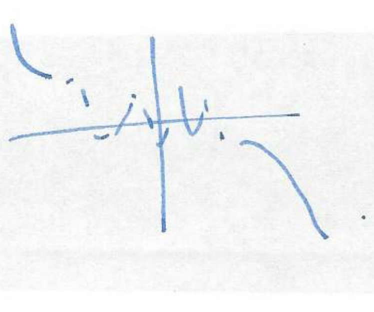
so $\quad p(x) \to +\infty$ as $x \to \infty$

$p(x) \to +\infty$ as $x \to -\infty$.

$p(x) = -4x^5 + x^3 - 3x^2 + 2$





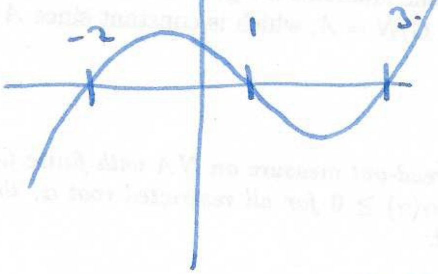


Using zeros to draw graph

Example $f(x) = (x+2)(x-1)(x-3)$.

	$x<-2$	$-2<x<1$	$1<x<3$	$x>3$
$(x+2)$	$-$	$+$	$+$	$+$
$(x-1)$	$-$	$-$	$+$	$+$
$(x-3)$	$-$	$-$	$-$	$+$
$f(x)$	$-$	$+$	$-$	$+$

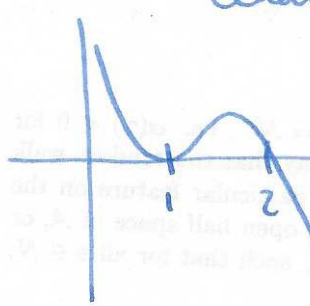
Leading term: x^3



Example $f(x) = (x-1)^2(2-x)$

	$x<1$	$1<x<2$	$x>2$
$(x-1)^2$	$+$	$+$	$+$
$(2-x)$	$+$	$+$	$-$
$f(x)$	$+$	$+$	$-$

Leading term $-x^3$.



Q: can you factor $x^4 - 2x^3 + 8x^2$ $\quad x^4 + 4x^2 + 3$.

Facts: a polynomial of degree n has at most n roots.

at most $n-1$ local min/max.