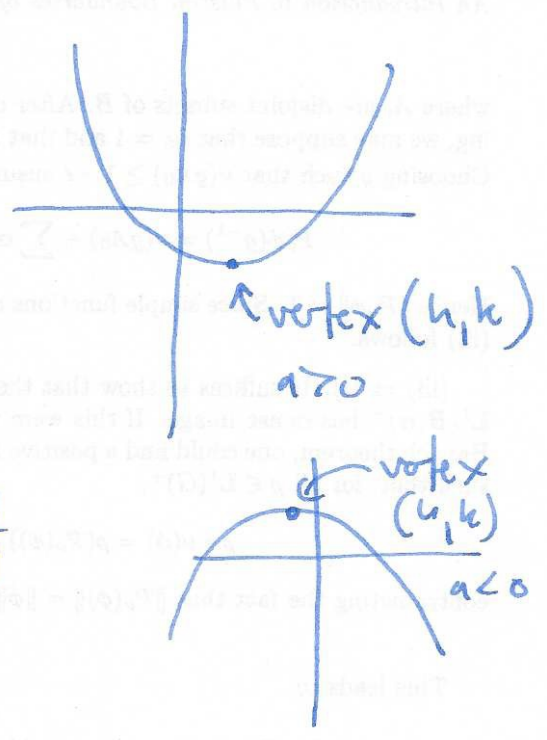


§ 3.1 Quadratic functions

$$f(x) = ax^2 + bx + c \quad (a \neq 0)$$

complete the square (standard form)

$$f(x) = a(x-h)^2 + k$$



Example

$$2x^2 + x - 1$$

$$2\left(x^2 + \frac{1}{2}x - \frac{1}{2}\right)$$

$$2\left(\left(x + \frac{1}{4}\right)^2 - \frac{9}{16}\right) = 2\left(x + \frac{1}{4}\right)^2 - \frac{1}{2}$$

$$2\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - \frac{1}{2}\right)$$

$$f(x) = a(x-h)^2 + k$$

$a > 0$ minimum value of f is $k = f(h)$

$a < 0$ maximum value of f is $k = f(h)$.

$$f(x) = a(x-h)^2 + k = a(x^2 - 2xh + h^2) + k = ax^2 - \frac{2ahx}{b} + \frac{ah^2 + k}{c}$$

$\therefore b = -2ah \Rightarrow h = -\frac{b}{2a}$ and $k = f\left(-\frac{b}{2a}\right)$.

W/k

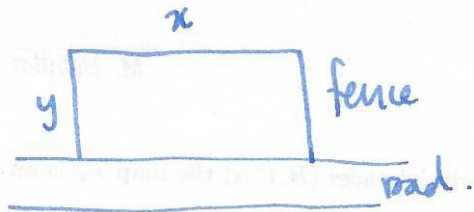
$\sin^2 x + \sin x + 2$ is quadratic in $\sin(x)$

$xy^3 + x^2y^2 + y^5$ is quadratic in x .

Examples

1. if you throw a ball upwards with velocity 40 ft/s then the height is given by $y = 40t - 16t^2$. What is the max height? When does it hit the ground?

2.



what is the largest area field you can enclose with 200 ft of fence?

$$\begin{aligned} \text{area} &= xy \\ \text{length} &= x + 2y = 200 \\ 2y &= 200 - x \\ y &= 100 - \frac{x}{2} \end{aligned}$$

$$\text{area} = x \left(100 - \frac{x}{2} \right) = -\frac{x^2}{2} + 100x$$

complete the square: $-\frac{1}{2} (x^2 - 200x)$

$$-\frac{1}{2} \left((x - 100)^2 - 10000 \right) = \frac{1}{2} (x - 100)^2 + 5000$$

$$(x^2 - 200x + 10000 - 10000)$$

solⁿ $x = 100$ $y = 50$ $\text{area} = 5000$.