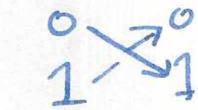
§2.7 One to one functions and their inverses

Example



one-to-one

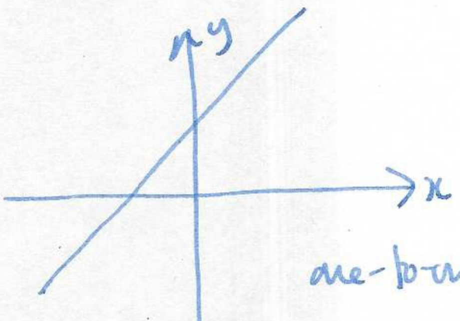
not one-to-one

Def^{n} A function $f: A \to B$ is called one-to-one if no two elements of A have the same image, i.e.

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

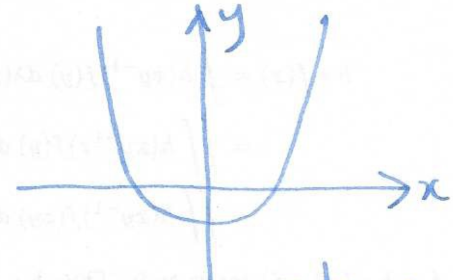
"different inputs give different outputs".

Example $f(x) = 2x+1$



one-to-one

$f(x) = x^2 - 1$.



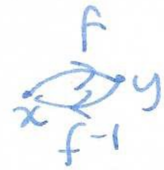
not one-to-one

Horizontal line test

A function is one-to-one if and only if no horizontal line intersects its graph more than once

If the function f is one-to-one, then it has an inverse f^{-1}.

$f: A \to B$ $\quad f^{-1}: B \to A$.



Def^{n} $f^{-1}(y) = x \Leftrightarrow f(x) = y$.

Example $f(x) = 2x+1$.

$f(1) = 3 \Rightarrow f^{-1}(3) = 1$

$f(-1) = -1 \Rightarrow f^{-1}(-1) = -1$.

<u>note</u> $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

$f(x) = 2x+1$ Q: what is a formula for f^{-1}?

$y = 2x+1$

$y-1 = 2x$

$\frac{y-1}{2} = x$ $\qquad f^{-1}(x) = \frac{x-1}{2}$.

<u>check!</u> $f(f^{-1}(x)) = x = f^{-1}(f(x))$.

$2\left(\frac{x-1}{2}\right)+1 = x-1+1 = x$

$\frac{(2x+1)-1}{2}$

$\frac{2x}{2} = x$.

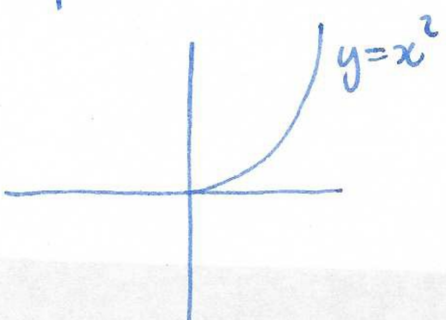
How to find a formula for the inverse:

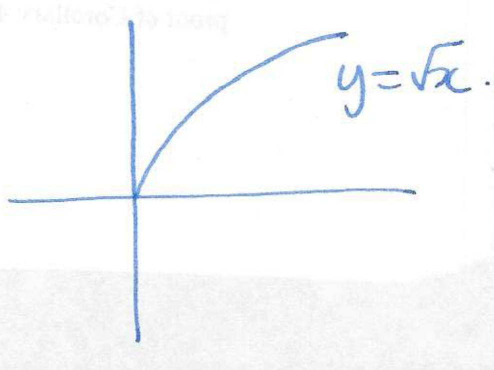
① solve $y = f(x)$ for x

② swap x and y.

<u>Useful fact</u>: the graph of $f^{-1}(x)$ is the reflection in $x=y$ of the graph of $f(x)$

<u>Example</u>





<u>Examples</u> find inverse of $\frac{1}{x+2}$ $\quad \frac{4x-2}{3x+1}$ $\quad \sqrt{2x-1}$.