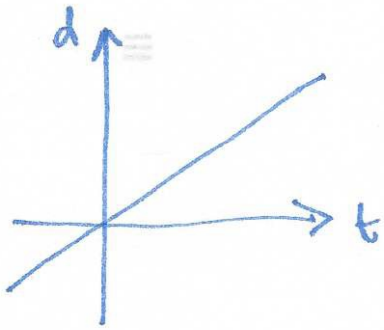
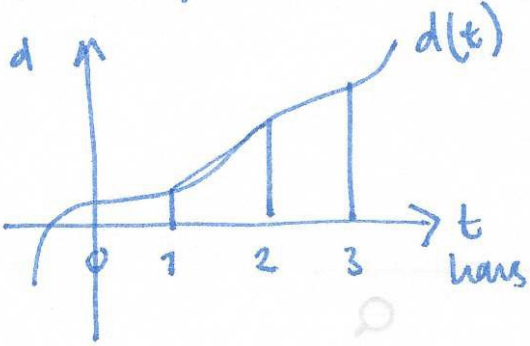
key fact: slope is rate of change.



$$\text{speed} = \frac{\text{distance}}{\text{time}}.$$

§2.4 Average rate of change

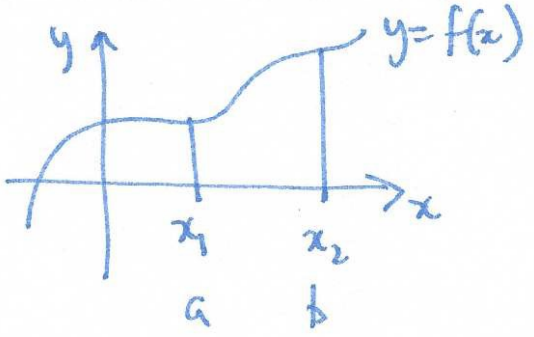
[For straight line functions average rate of change = rate of change!]



$$\text{average rate of change} = \text{average speed} = \frac{\text{change in distance}}{\text{change in time}}$$

average rate of change between $t=1$ and $t=2$ is $\dfrac{d(2)-d(1)}{2-1}$.

in general



average rate of change between x_1 and x_2 is $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

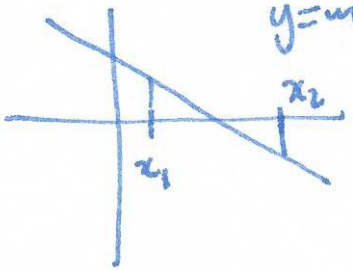
Example $f(x) = (x-2)^2$ $\quad a=4, b=3.$

Example falling objects: $d(t) = 16t^2$

Q: · how far do you fall in 1 sec
2 sec
3 sec

· what is aveage rat speed between 1 and 2, 2 and 3?

Linear functions have constant rate of change:

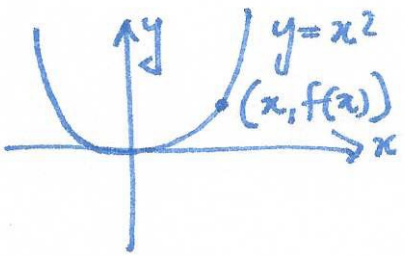


$$\text{rate of change} = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{mx_2+b-(mx_1+b)}{x_2-x_1}$$

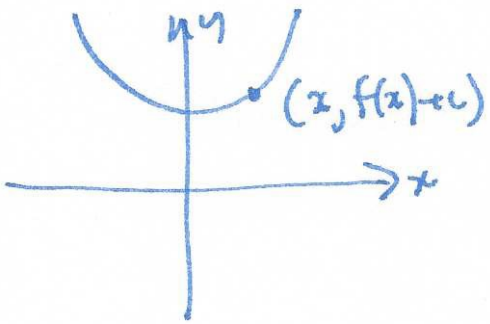
$$= \frac{mx_2-mx_1}{x_2-x_1} = m\,\frac{x_2-x_1}{x_2-x_1} = m.$$

§2.5 Transformations of functions

suppose we know graph of $y=f(x)$

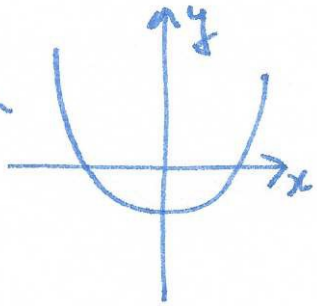


Q: what does graph of $f(x)+c$ look like?

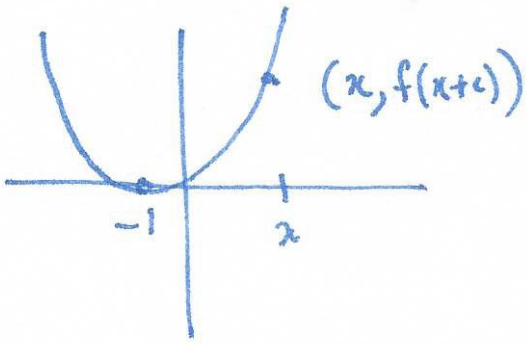


$c>0$ vertical shift up

$c<0$ vertical shift down



Q: what does graph of $f(x+c)$ look like?



Example: $f(x) = x^2$

$f(x+1) = (x+1)^2 = x^2+2x+1$

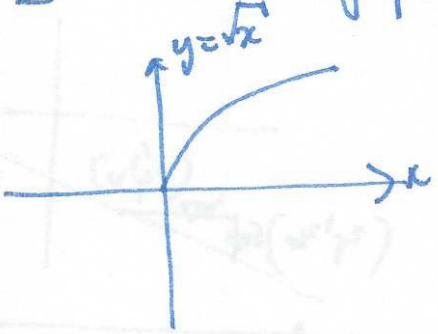
$c>0$ shift to the left.

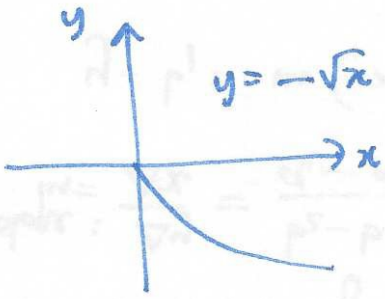
$c<0$ shift to the right

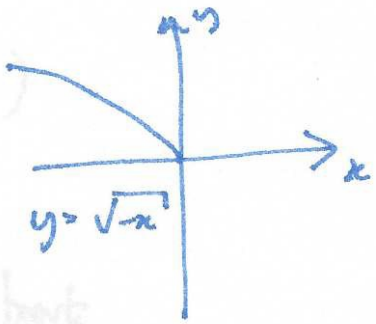
Example $f(x) = \sqrt{x}$

draw graph of $f(x) = \sqrt{x-2}+3$.

Q: what do graphs of $-f(x)$ and $f(-x)$ look like?





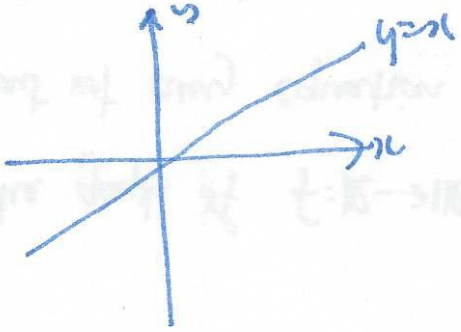


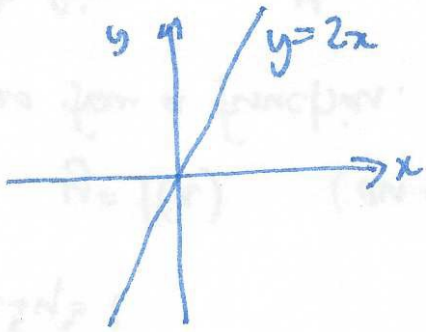
$-f(x)$: reflection in x-axis

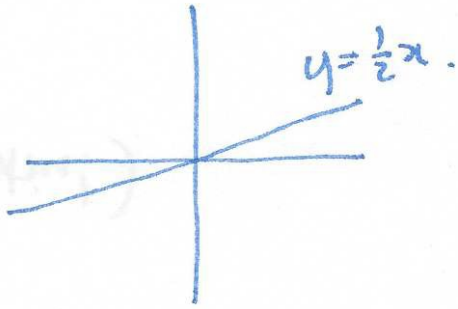
$f(-x)$: reflection in y-axis

Example x^3+x+1?

Q: what does graph of $y=cf(x)$ look like?





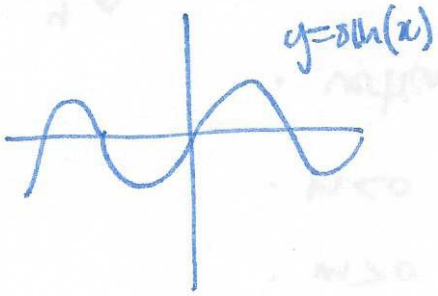


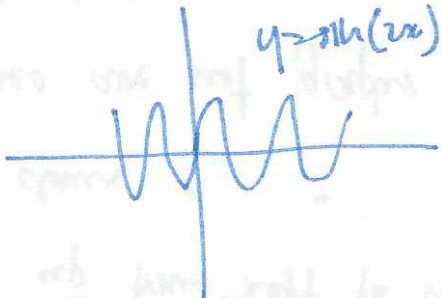
$cf(x)$: vertical scale by factor of c: $c>1$ expand

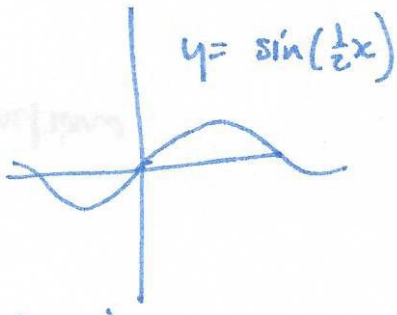
$0<c<1$ shrink

Q: $-2f(x)$?

Q: what does graph of $f(cx)$ look like?







$f(cx)$ horizontal scale by factor of c: $c>1$ shrink

$0<c<1$ expand.