Math 130 Pre Calculus Section 17977

Joseph Maher joseph.maher@csi.cuny.edu

web page: http://www.math.csi.cuny.edu/~maher

office 15-222 office hours M 12:20 – 2:15
W 1:25 – 2:15

- math tutoring: 15-214
- students with disabilities

Text: Precalculus, Stewart, Redlin, Watson, 6th ed.

WebAssign: csi.cuny 0155 4835 www.webassign.net

§2.1 Functions and domains

Example USPS letter postage:

weight (oz) w	$w<1$	$1<w<2$	$2<w<3$	$3<w<3.5$
cost (¢)	0.49	0.70	0.91	1.12

- your height function of time/age.
- temperature function of time.

Def^n need 3 things: name/rule f
input/domain A
output/range B

notation: $f: A \longrightarrow B$ (name, domain, range)

$x \longmapsto f(x)$

key property: for each $x \in A$, there is exactly one $f(x) \in B$.

Example $f: [0, 3.5] \longrightarrow \mathbb{R}$

interval: all numbers $x \in \mathbb{R}$ with $0 \le x \le 3.5$

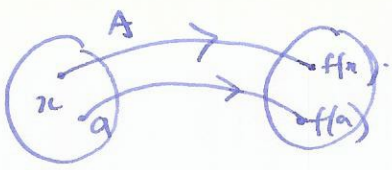
$$\text{cost}(w) = f(w) = \begin{cases} 0.49 & \text{if } w < 1 \\ 0.70 & \text{if } 1 \le w < 2 \\ 0.91 & \text{if } 2 \le w < 3 \\ 1.12 & 3 \le w < 3.5 \end{cases}$$

Note: weight is not a function of cost!

Different ways to think of a function:

- rule/machine $x \longmapsto \boxed{f} \longmapsto f(x)$ (input, output) e.g.: $\sqrt{x}$ on your calculator

- arrow diagram

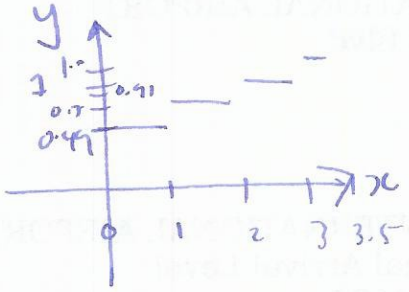


e.g. {people} → {height} {favourite colour}

{Alice} ↦ 5'2" red

{Bob} ↦ 5'4" green

- graph $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto f(x)$



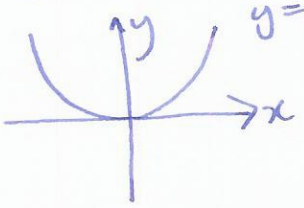
y = cost (weight).

draw graph $y = f(x)$

notation: y ↑ dependent variable; x ↑ (independent) variable

Note: ~~name of dependent~~ Example function given by formula.

e.g. $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$ or $f(x) = x^2$ can draw graph



$y = f(x) = x^2$

Note: name of dependent variable doesn't matter.

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ } same function!

$f: \mathbb{R} \to \mathbb{R}$, $f(t) = t^2$ }

Defn: function $f: A \to B$ Q: what do we mean by rule?

Don't need a formula/description, just a subset of $A \times B$ such that for each $x \in A$ there is ~~at most one~~ exactly one $(x, b) \in A \times B$.

Example {Alice, Bob} → {red, green}

A B.

$A \times B$ = {(Alice, red), (Alice, green), (Bob, red), (Bob, green)}

In this course lots of functions will be given by formulae.

Example $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 4x^2 - 12x + 3$.

formulas are useful because we can evaluate them.

$$f(2) = 4(2)^2 - 12(2) + 3 = \overset{16}{4\times4} \overset{-24}{-12\cdot2} \overset{+3}{+3} = -5.$$

$$f(a+b) = 4(a+b)^2 - 12(a+b) + 3 = 4(a^2+2ab+b^2) - 12(a+b) + 3$$

$$= 4a^2 + 8ab + 4b^2 - 12a - 12b + 3.$$

$f(x-2x^2)$?

Warning : sometimes people just give a formula, without domain or range

remember : $f: A \to B$.

$f(x) = \sqrt{x}$ ← you are meant to work out domain and range

$f: \mathbb{R} \to \mathbb{R}$

$[0, \infty)$.

Example $f(x) = \frac{1}{x^2 - 2x}$ find domain.

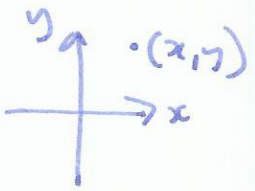
§2.2 Graphs of functions

$A \to B$.

$f: \mathbb{R} \to \mathbb{R}$ the graph of f is the set of pairs $(x, f(x))$

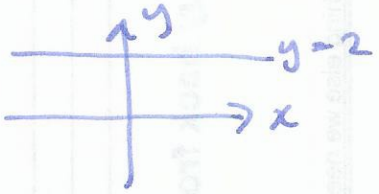
$x \mapsto f(x)$

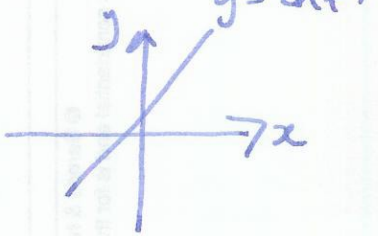
If $f: \mathbb{R} \to \mathbb{R}$ then this is a subset of the plane $\mathbb{R}^2$

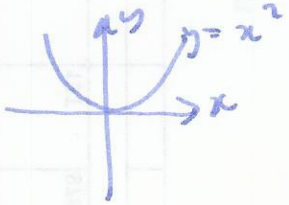


Examples

- constant function $f(x) = 2$
- straight line function $f(x) = 2x + 1$
- quadratic function $f(x) = x^2$

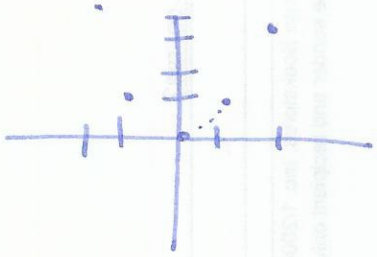






Plotting by hand: make a table

x	-2	-1	0	1	2
$f(x) = x^2$	4	1	0	1	4



more detail

x	0	$1/4$	$1/2$	$3/4$	1
x^2	0	$1/16$	$1/4$	$9/16$	1