Math 130 Precalculus Fall 14 Midterm 3a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Solve

(a) $3e^{2x} + 2e^x = 1$

$$3(e^x)^2 + 2e^x - 1 = 0$$

$$(3e^x - 1)(e^x + 1) = 0$$

$$e^x = 1/3 \qquad \underbrace{e^x = -1}_{\text{no solutions}}$$

$$x = \ln(1/3)$$
$$= -\ln(3)$$

(b) $\ln(x) = \ln(3x-1) + 1$

$$\ln\left(\frac{x}{3x-1}\right) = 1$$

$$\frac{x}{3x-1} = e$$

$$x = 3xe - e$$

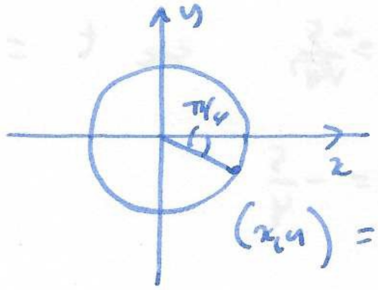
$$x(1-3e) = -e$$

$$x = \frac{-e}{1-3e}$$

(2) Find the point on the unit circle

(a) corresponding to the terminal point for $t = -25\pi/6$.

$\frac{-25}{6} = -4 - \frac{1}{6}$



$(x,y) = \left(\cos\left(-\frac{\pi}{6}\right), \sin\left(-\frac{\pi}{6}\right)\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

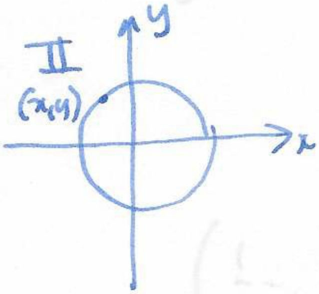
(b) whose y-coordinate is $-2/5$ and whose x-coordinate is positive.

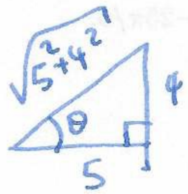
$x^2 + y^2 = 1$

$x^2 + \frac{4}{25} = 1 \qquad x^2 = \frac{21}{25} \qquad x = \pm \frac{\sqrt{21}}{5}$

$x = +\frac{\sqrt{21}}{5}$

(3) If $\tan(t) = -4/5$ and t is in quadrant II find the values of the other trig functions at t.





$\sin t = \frac{4}{\sqrt{41}}$ $\quad$ $\operatorname{cosec} t = \frac{\sqrt{41}}{4}$

$\cos t = \frac{-5}{\sqrt{41}}$ $\quad$ $\sec t = -\frac{\sqrt{41}}{5}$

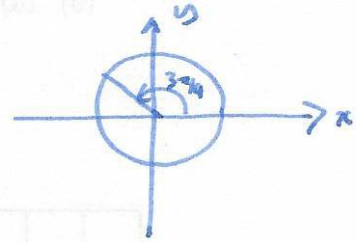
$\cot(t) = -\frac{5}{4}$

$25+16 = 41$

(4) Find the exact value of

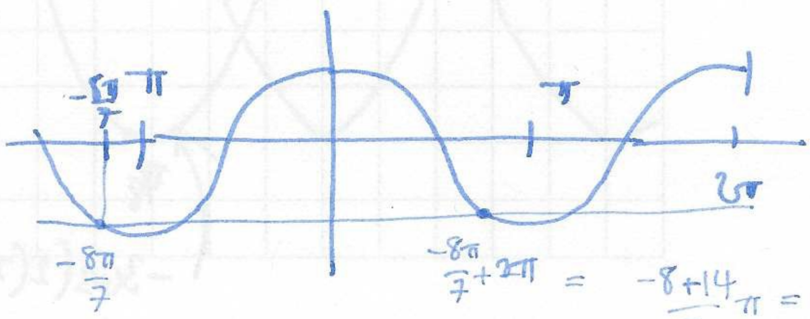
(a) $\tan(23\pi/4)$

$$\frac{23}{4} = 5 + \frac{3}{4}$$



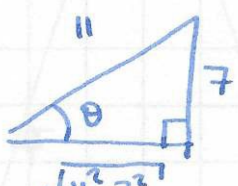
$$= \tan\left(\frac{3\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1$$

(b) $\cos^{-1}(\cos(-8\pi/7))$



$$\frac{-8\pi}{7} + 2\pi = \frac{-8+14}{7}\pi = \frac{6}{7}\pi$$

(c) $\tan(\sin^{-1}(7/11))$

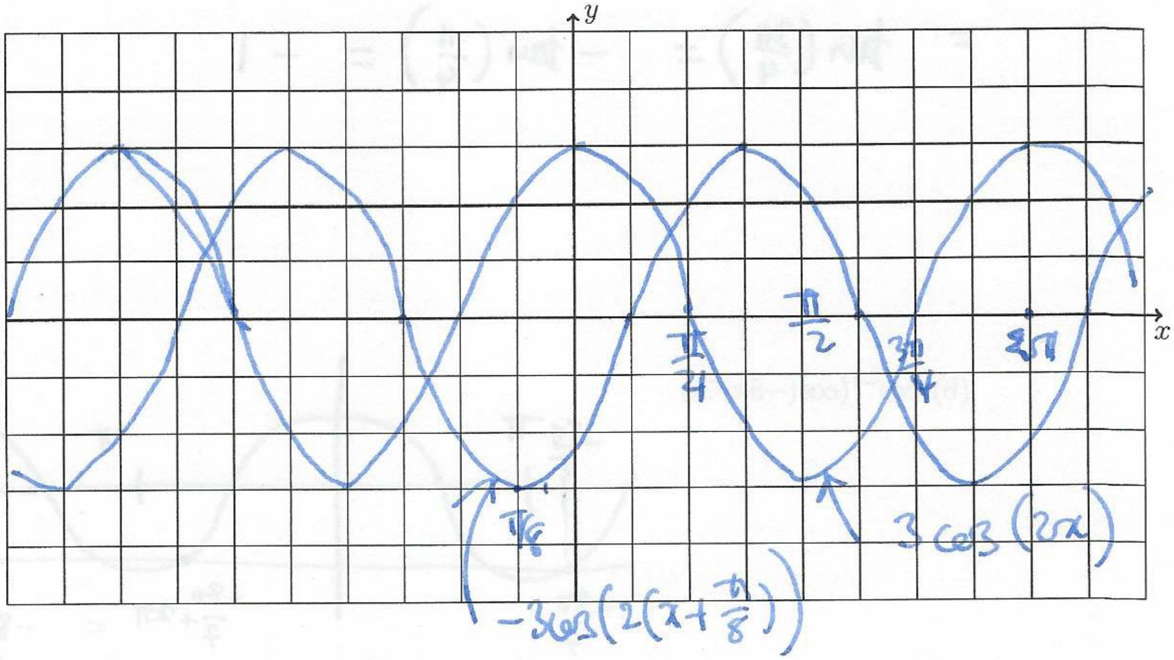


$$\tan\theta = \frac{7}{\sqrt{72}}$$

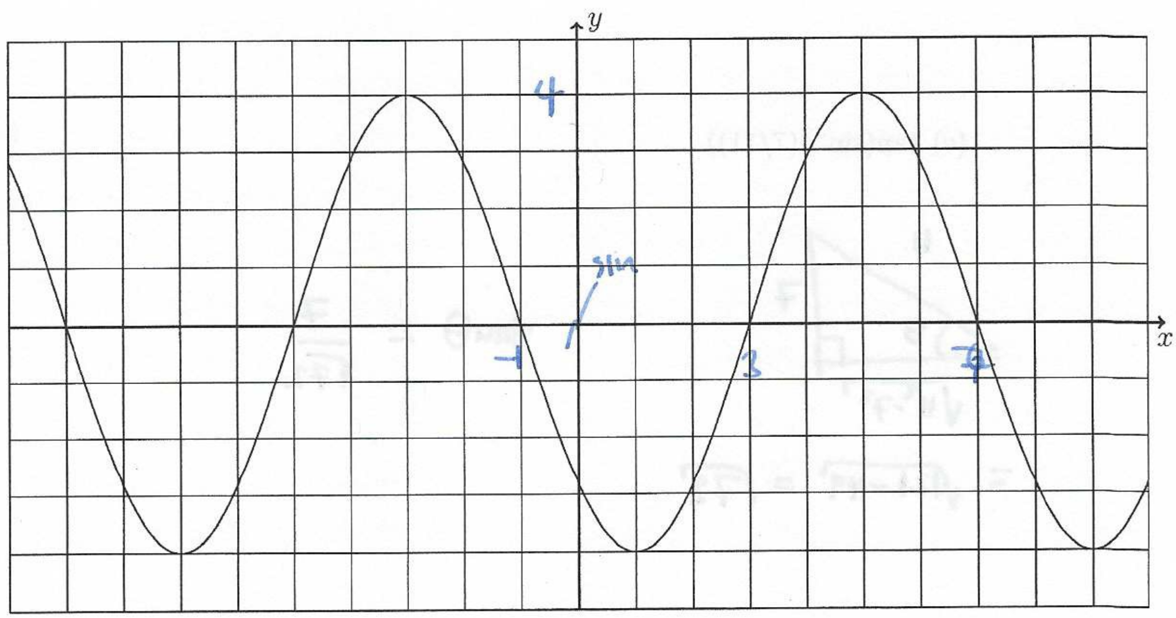
$$= \sqrt{121-49} = \sqrt{72}$$

(5) (a) Find the amplitude, frequency and phase shift for $y = -3\cos(2x + \pi/4)$, and draw a careful graph of the function below.

$y = -3\cos\left(2\left(x + \frac{\pi}{8}\right)\right)$



(b) Find an equation describing the graph below.



period $= 8$

$= \frac{2\pi}{k}$

$k = \frac{\pi}{4}$

$a = 4$

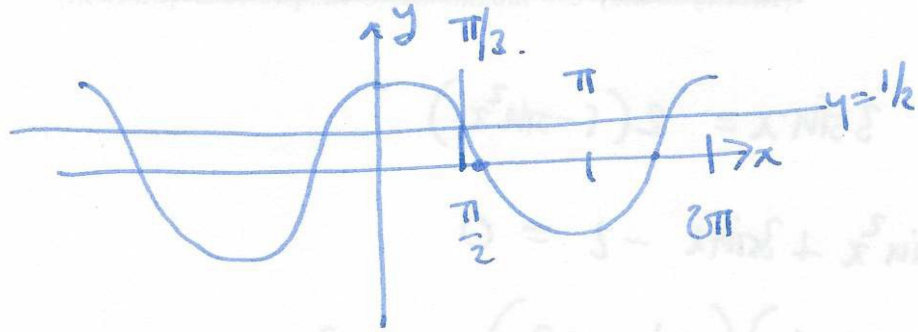
$4\sin\left(\frac{\pi}{4}(x-b)\right)$

$4\sin\left(\frac{\pi}{4}(x-3)\right)$.

(6) Find all solutions to

$$2\cos(x) = 1.$$

$\cos(x) = \frac{1}{2}$



$x = \frac{\pi}{3} + 2\pi n$

$n \in \mathbb{Z}$

$x = -\frac{\pi}{3} + 2\pi n$

(7) Find all solutions to

$$3\sin(x) = 2\cos^2(x).$$

~~(Hint: you may use the answer to the previous question.)~~

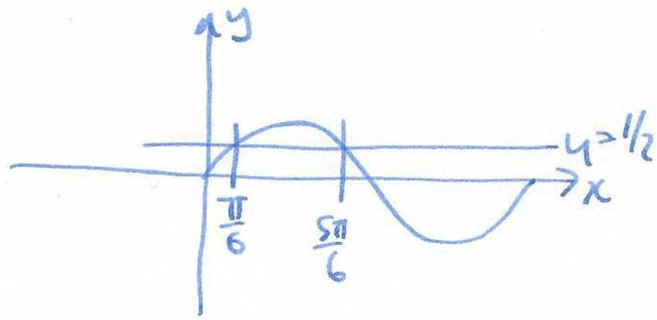
$3\sin x = 2(1-\sin^2 x)$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = -2$ ~~need~~ no solutions.

$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6} + 2\pi n$

$x = \frac{5\pi}{6} + 2\pi n$

(8) Find all solutions to

$$\sin(x) + \sin(2x) = 0.$$

$$\sin x + 2\sin x \cos x = 0$$

$$\sin x\,(1 + 2\cos x) = 0$$

$\sin x = 0$

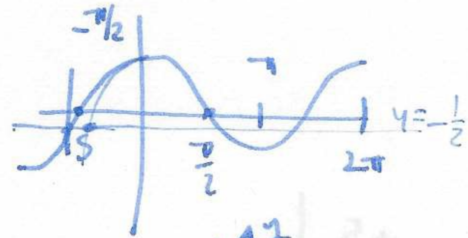
$x = 0 + 2\pi n$

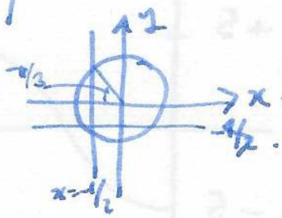
$\quad \pi + 2\pi n$

$\cos x = -1/2$

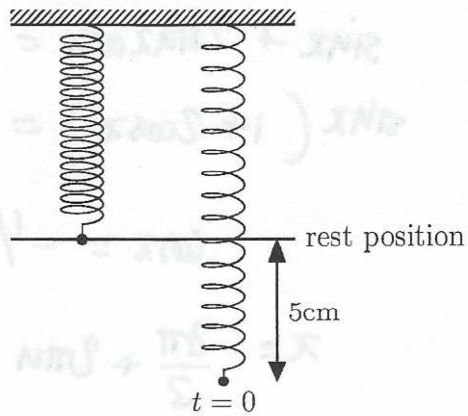
$x = \dfrac{2\pi}{3} + 2\pi n$

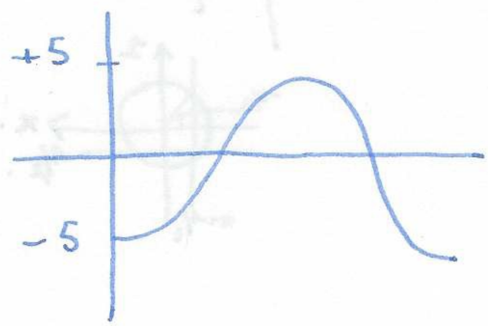
$\quad -\dfrac{2\pi}{3} + 2\pi n$





(9) A spring is pulled down 5cm from its rest position and then released. If it moves according to simple harmonic motion and takes $\frac{1}{4}$ of a second to reach its highest point, find an equation for the height of the spring.





$$\text{period} = \frac{1}{2} = \frac{2\pi}{k}$$

$$k = 4\pi$$

$$-5\cos(4\pi t)$$

(10) Prove the identity

$$\frac{\tan x}{\sec x - 1} = \frac{\sec x + 1}{\tan x}.$$

$$\frac{\tan x}{\sec x - 1} \; \frac{\sec x + 1}{\sec x + 1} = \frac{\tan x\,(\sec x + 1)}{\sec^2 x - 1} = \frac{\tan x\,(\sec x + 1)}{\tan^2 x}$$

use: $\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$$= \frac{\sec x + 1}{\tan x}.$$