

Sample midterm 3 solutions

Q1 $\log_5(3) = \frac{\ln(3)}{\ln(5)} = \boxed{0.613}$

Q2 a) $\ln(2x+3) = \ln(x) + 4$
 $\ln\left(\frac{2x+3}{x}\right) = 4$
 $2 + \frac{3}{x} = e^4$

$\frac{3}{x} = e^4 - 2$
 $x = \boxed{\frac{3}{e^4 - 2}}$

b) $2e^x - 3e^{-x} = 2$
 $2e^x - 2 - 3e^{-x} = 0$
 $2(e^x)^2 - 2e^x - 3 = 0$

$e^x = \frac{2 \pm \sqrt{4+24}}{4} = \frac{1 \pm \sqrt{7}}{2}$
 $e^x = \frac{1+\sqrt{7}}{2} \Rightarrow \boxed{x = \ln\left(\frac{1+\sqrt{7}}{2}\right)}$
 $e^x = \frac{1-\sqrt{7}}{2} < 0$ no solutions.

c) $\frac{3}{1+e^{-2x}} = 1$
 $3 = 1 + e^{-2x}$
 $e^{-2x} = 2$
 $-2x = \ln(2)$
 $x = -\frac{1}{2}\ln(2)$

d) $4 \tan x - \sec^2 x = 0$

use: $\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$4 \tan x - 1 - \tan^2 x = 0$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x - 4 \tan x + 1 = 0$

$\tan x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$

$x = \tan^{-1}(2 + \sqrt{3}), \tan^{-1}(2 - \sqrt{3})$

e) $\sin(2x) \cos(x) - \cos(2x) \sin(x) = 0$

use: sine addition formula.

$\sin(2x - x) = \sin(x) = 0$

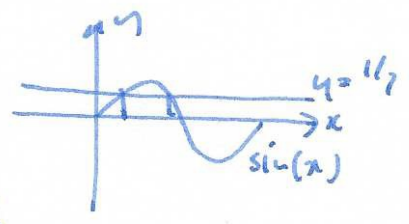
$x = 0 + 2\pi u, \pi + 2\pi u, u \in \mathbb{Z}$

f) $\sin(4x) - \cos(2x) = 0$

use: double angle

$2 \sin(2x) \cos(2x) - \cos(2x) = 0$

$\cos(2x) (2 \sin(2x) - 1) = 0$



solutions: $\cos(2x) = 0$

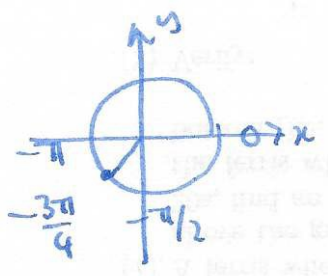
$\sin(2x) = 1/2$

$2x = \pi/6 + 2\pi u, 5\pi/6 + 2\pi u$

$x = \pi/12 + \pi u, 5\pi/12 + \pi u, u \in \mathbb{Z}$

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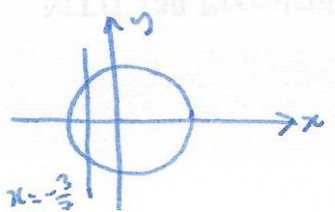
Q3 a)



$-\frac{11\pi}{4} = -2\pi - \frac{3\pi}{4}$

terminal point $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

b)



$x^2 + y^2 = 1$

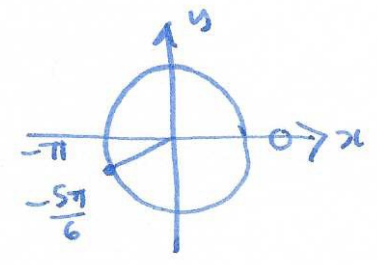
$y^2 = 1 - \frac{9}{49} = \frac{40}{49}$

$(-\frac{3}{7})^2 + y^2 = 1$

$y = \frac{\sqrt{40}}{7}$

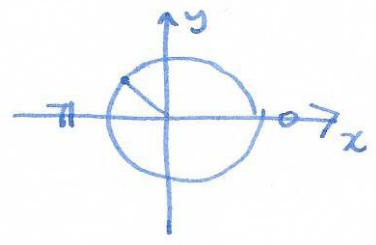
Q4 a) $\sec\left(-\frac{17\pi}{6}\right)$ $\text{ux: } -\frac{17\pi}{6} = -2\pi - \frac{5\pi}{6}$

$$= \frac{1}{\cos\left(-\frac{5\pi}{6}\right)} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$$



b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$$= \pi - \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$$



c) $\sin^{-1}\left(\sin\left(-\frac{5\pi}{3}\right)\right)$

$$= \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$$

$\text{ux: } -\frac{5\pi}{3} + 2\pi = \frac{\pi}{3}$

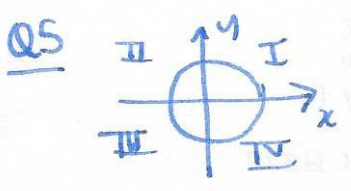
d) $\sec\left(\tan^{-1}\left(\frac{3}{2}\right)\right)$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}} = \boxed{\frac{\sqrt{13}}{2}}$$

e) $\cos\left(\frac{5\pi}{12}\right)$ $\text{ux: } \frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$

$$\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$



$$\frac{\sin^2 t + \cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}$$

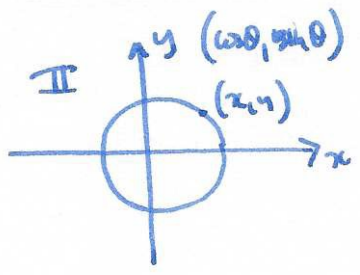
$$\cot^2 t = \frac{1}{\sin^2 t} - 1$$

$$1 + \cot^2 t = \frac{1}{\sin^2 t}$$

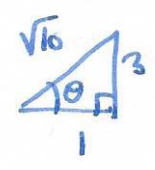
$$\cot t = \pm \sqrt{\frac{1}{\sin^2 t} - 1}$$

$$\boxed{\cot t = -\sqrt{\frac{1}{\sin^2 t} - 1}}$$

Q6



$\tan t = -3$



$\cot t = -\frac{1}{3}$

(4)

$\cos t = -\frac{1}{\sqrt{10}}$

$\sec t = -\sqrt{10}$

$\sin t = \frac{3}{\sqrt{10}}$

$\csc t = +\frac{\sqrt{10}}{3}$

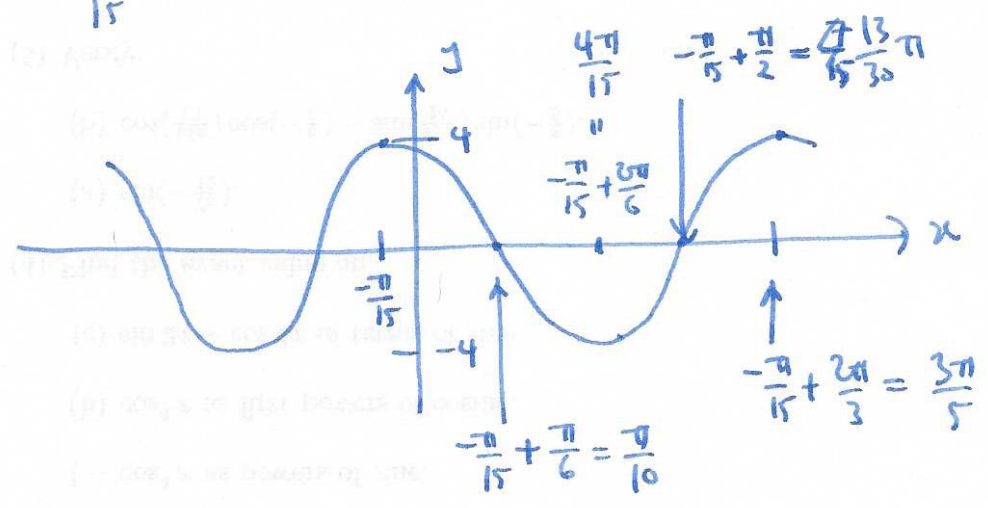
Q7

$y = 4 \cos\left(3x + \frac{\pi}{5}\right) = 4 \cos\left(3\left(x - \left(-\frac{\pi}{15}\right)\right)\right)$

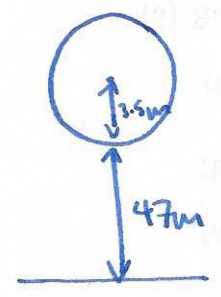
amplitude = 4

period = $\frac{2\pi}{3}$ frequency = $\frac{1}{\text{period}} = \frac{3}{2\pi}$

phase shift = $-\frac{\pi}{15}$



Q8



$h = 3.5 \cos(2\pi t) + 47 + 3.5$

period = $\frac{2\pi}{k} = 1 \text{ hour}$ $k = 2\pi$

$h = 3.5 \cos(2\pi t) + 50.5$

Q9

$y = 4 \cos\left(\frac{\pi}{4}(x - (-1))\right) = 4 \cos\left(\frac{\pi}{4}(x+1)\right)$

period = $8 = \frac{2\pi}{k}$ $k = \frac{\pi}{4}$

Q10 a)
$$\frac{1 + \cos(x)}{\sin(x)} \cdot \frac{1 - \cos(x)}{1 - \cos(x)} = \frac{1 - \cos^2(x)}{\sin(x)(1 - \cos(x))} = \frac{\sin^2(x)}{\sin(x)(1 - \cos(x))} = \frac{\sin(x)}{1 - \cos(x)}$$

b)
$$\tan(x) + \cot(x) = \frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)} = \frac{\sin^2(x) + \cos^2(x)}{\cos(x)\sin(x)} = \frac{1}{\cos(x)\sin(x)} = \sec(x)\csc(x)$$

c)
$$2\csc(2x)\tan(x) = \frac{2}{\sin(2x)} \frac{\sin(x)}{\cos(x)} = \frac{2}{2\sin(x)\cos(x)} \frac{\sin(x)}{\cos(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$