

# Math 130 Precalculus Fall 14 Midterm 2b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

maximum

- (1) (10 points) Find the ~~minimum~~ value of the function  $f(x) = 3x + 1 - 2x^2$  by completing the square.

$$\begin{aligned}
 -2x^2 + 3x + 1 &= -2\left(x^2 - \frac{3}{2}x\right) + 1 \\
 &= -2\left(\left(x - \frac{3}{4}\right)^2 - \frac{9}{16}\right) + 1 = -2\left(x - \frac{3}{4}\right)^2 + 1 + \frac{9}{8} \\
 &= -2\left(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) + 1
 \end{aligned}$$

max value is  $1 + \frac{9}{8} = \frac{17}{8}$

1	10
2	10
3	10
4	10
5	10
6	10
7	10
8	10
9	10
10	10
30	

Maximum	
Grade	

$$\frac{(x+1)^2}{(x+1)(x+4)} = \frac{x+1}{x+4}$$

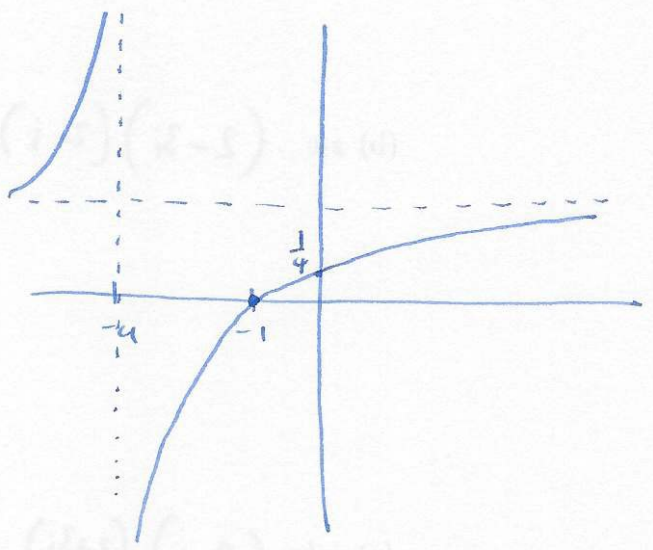
(2) (10 points) Sketch the graph of the function  $f(x) = \frac{(x+1)^2}{x^2+5x+4}$ . Include the x- and y-intercepts, and the vertical and horizontal asymptotes.

y intercept:  $f(0) = \frac{1^2}{4} = \frac{1}{4}$       x-intercepts:  $(x+1)^2 = 0 \Rightarrow x = -1$

vertical asymptotes:  $x^2+5x+4 = (x+4)(x+1) \Rightarrow x = -4, -1$       horizontal asymptotes:  $\frac{x^2}{x^2} = 1$

sign:

$(x+1)^2$	+	+	+
$(x+4)$	-	+	+
$(x+1)$	-	-	+
$f(x)$	+	-	+



(3) (10 points) Let  $z = 2 - 3i$  and let  $w = 2 - i$ . Write the following complex numbers in the form  $a + bi$ , where  $a$  and  $b$  are real numbers.

(a)  $w - 2z$

$$\begin{aligned} 2 - i - 2(2 - 3i) &= 2 - i - 4 + 6i \\ &= -2 + 5i \end{aligned}$$

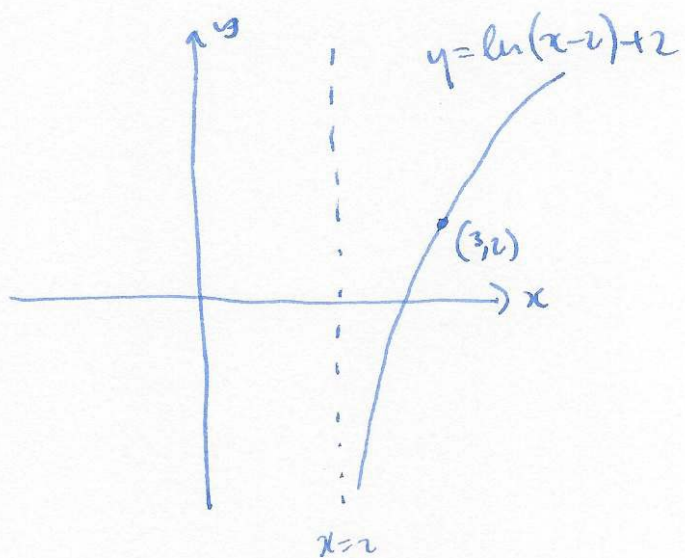
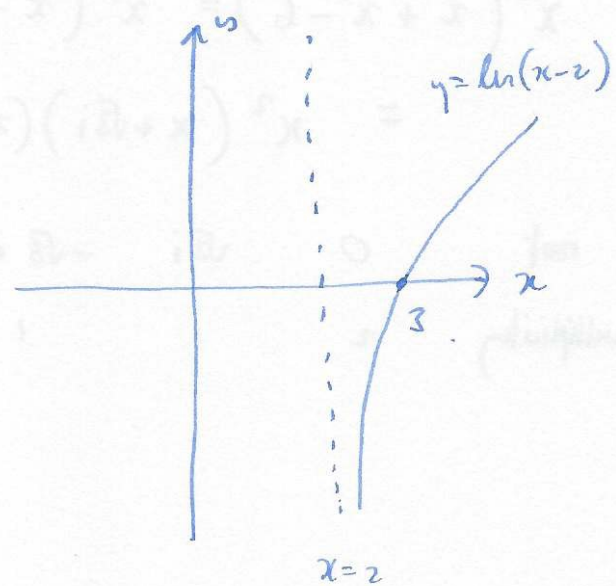
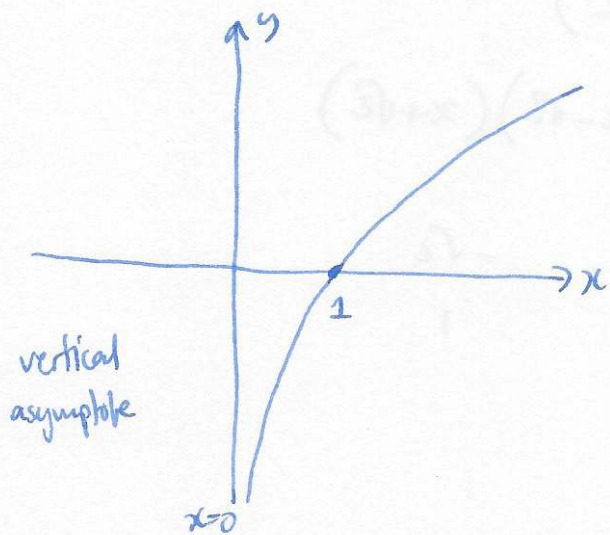
(b)  $zw$

$$\begin{aligned} (2 - 3i)(2 - i) &= 4 - 2i - 6i - 3 \\ &= 1 - 8i \end{aligned}$$

(c)  $w/z$

$$\begin{aligned} \frac{(2 - i)(2 + 3i)}{(2 - 3i)(2 + 3i)} &= \frac{4 + 6i - 2i + 3}{4 + 9} = \frac{7 + 4i}{13} \\ &= \frac{7}{13} + \frac{4}{13}i \end{aligned}$$

- (4) (10 points) Sketch the graph of  $f(x) = \ln(x - 2) + 2$ , including any vertical or horizontal or vertical asymptotes the graph has, if any.



(5) (10 points) Find all roots of the polynomial  $x^6 + x^4 - 6x^2$ , and state their multiplicities.

$$x^2(x^4 + x^2 - 6) = x^2(x^2 + 3)(x^2 - 2)$$

$$= x^2(x + \sqrt{3}i)(x - \sqrt{3}i)(x - \sqrt{2})(x + \sqrt{2})$$

root	0	$\sqrt{3}i$	$-\sqrt{3}i$	$\sqrt{2}$	$-\sqrt{2}$
multiplicity	2	1	1	1	1

- (6) (10 points) Use the quadratic formula to find all roots of  $f(x) = 3x^2 + 2x + 5$ , expressing them in the form  $a + bi$ , where  $a$  and  $b$  are real numbers.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4 \times 3 \times 5}}{6}$$

$$x = \frac{-2 \pm \sqrt{4 - 60}}{6}$$

$$x = \frac{-2 \pm \sqrt{-56}}{6}$$

$$x = -\frac{1}{3} + \frac{\sqrt{56}}{6}i, -\frac{1}{3} - \frac{\sqrt{56}}{6}i$$

$$x = -\frac{1}{3} + \frac{\sqrt{14}}{3}i, -\frac{1}{3} - \frac{\sqrt{14}}{3}i$$

(7) (10 points) Use the log rules to:

(a) Evaluate  $\log_3(9)$ .

2

(b) Expand  $\ln(y^2/\sqrt{x})$ .

$$2\ln(y) - \frac{1}{2}\ln(x)$$

(c) Combine  $\log_2(x+2) - \frac{1}{3}\log_2(x+1)$  as a single logarithm.

$$\log_2\left(\frac{x+2}{\sqrt[3]{x+1}}\right)$$



(8) (10 points) Solve the following equations.

(a)  $e^{3x+2} = 4$

$$3x+2 = \ln(4)$$

$$x = \frac{\ln(4) - 2}{3}$$

(b)  $\log_4(x+5) - \log_4(x-1) = 2$

$$\log_4 \left( \frac{x+5}{x-1} \right) = 4^2$$

$$\frac{x+5}{x-1} = 4^2 = 16$$

$$x+5 = 16x-16$$

$$21 = 15x$$

$$x = \frac{21}{15}$$

(9) (10 points) You put \$400 in a bank account with 5% interest per year.

(a) If the interest is compounded monthly, how much will you have after 6 years?

$$P \left( 1 + \frac{r}{n} \right)^{nt} \quad 400 \left( 1 + \frac{0.05}{12} \right)^{12 \times 6} \approx 539.61$$

(b) If the interest is compounded continuously, how much will you have after 6 years?

$$Pe^{rt} \quad 400 e^{0.05 \times 6} \approx 539.94$$

- (10) (10 points) You put \$400 in a bank account with 5% interest per year. If interest is compounded continuously, how long will it be before you have \$600?

Pert

$$400 e^{0.05t} = 600$$

$$e^{0.05t} = \frac{3}{2}$$

$$0.05t = \ln(3/2)$$

$$t = \frac{\ln(3/2)}{0.05} \approx 8.11 \text{ years.}$$