Math 130 Precalculus Fall 14 Midterm 1b

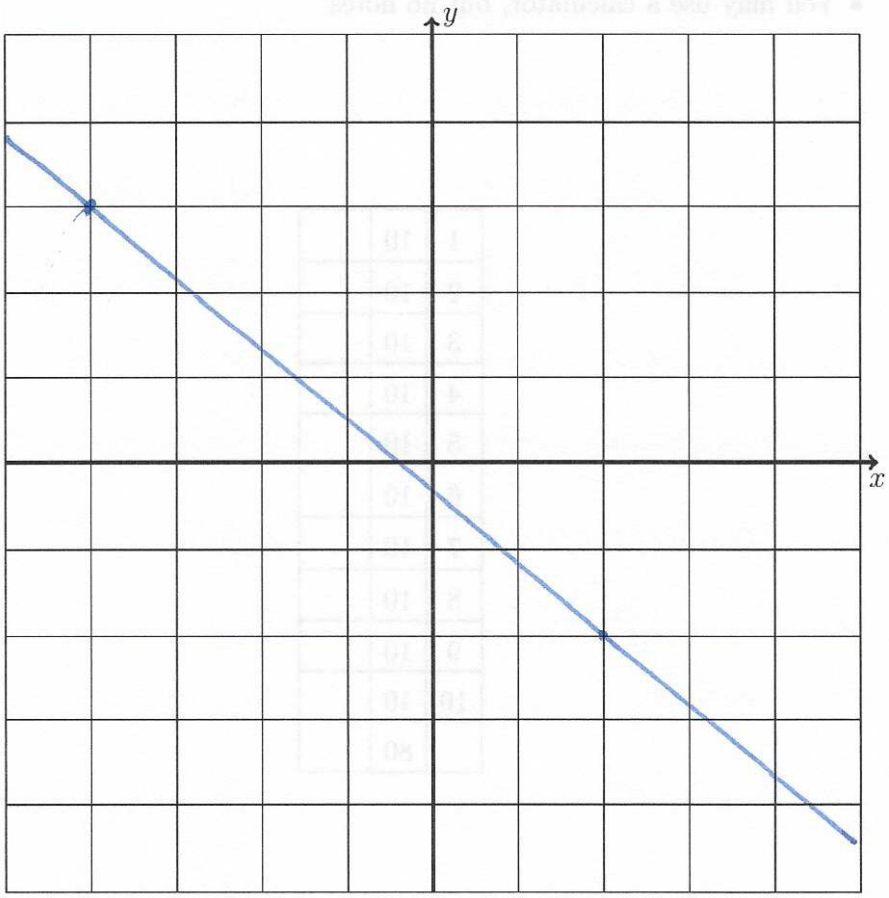
Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-4, 3)$ and $(2, -2)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



slope $\frac{-2-3}{2-(-4)} = \frac{-5}{6}$

$$\boxed{y-3 = -\frac{5}{6}(x+4)}$$

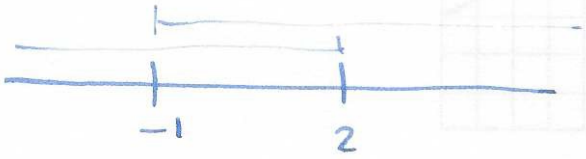
$$y = -\frac{5}{6}x - \frac{10}{3} + 3$$

$$y = -\frac{5}{6}x - \frac{1}{3}$$

(2) (10 points) Find the domain of the function $f(x) = \sqrt{x+1} + \sqrt{2-x}$.

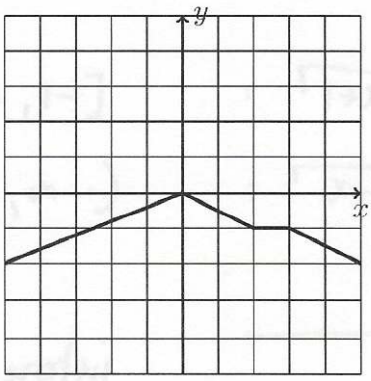
domain of $\sqrt{x+1}$: $[-1, \infty)$

$\sqrt{2-x}$: $(-\infty, 2]$



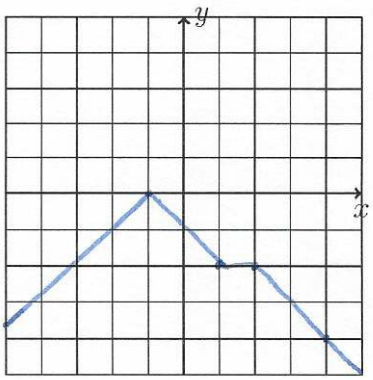
intersection: $\boxed{[-1, 2]}$

(3) (10 points) The graph of the function $f(x)$ is given below.



$f(x)$

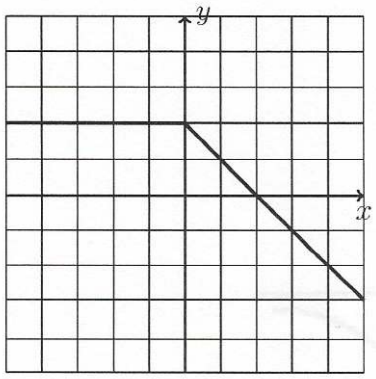
(a) Sketch the graph obtained from the graph of $f(x)$ by expanding it by a factor of 2 in the vertical direction, and then shifting it 1 unit to the left.



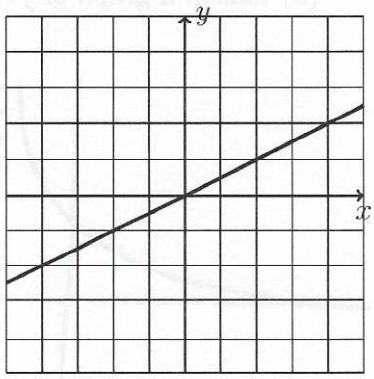
(b) Write down a formula for the function you have drawn above, expressed in terms of $f(x)$. Do not attempt to find a formula for $f(x)$ in terms of x.

$2f(x+1)$

(4) (10 points) The graphs of the functions $f(x)$ and $g(x)$ are shown below.

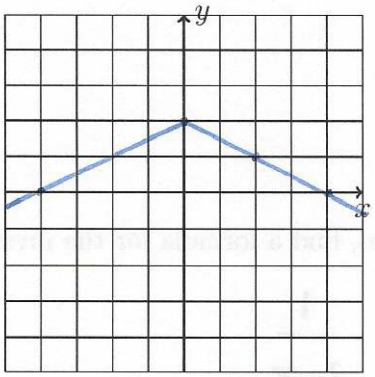


$f(x)$

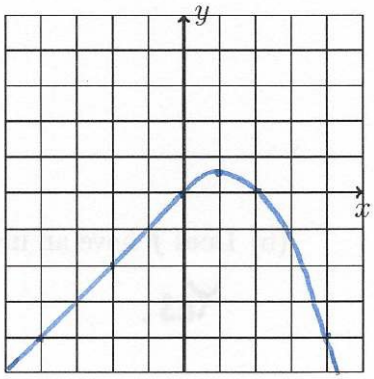


$g(x)$

(a) Sketch graphs of the following functions.



$f(x) + g(x)$



$f(x)g(x)$

(b) Find $(f \circ g)(4)$.

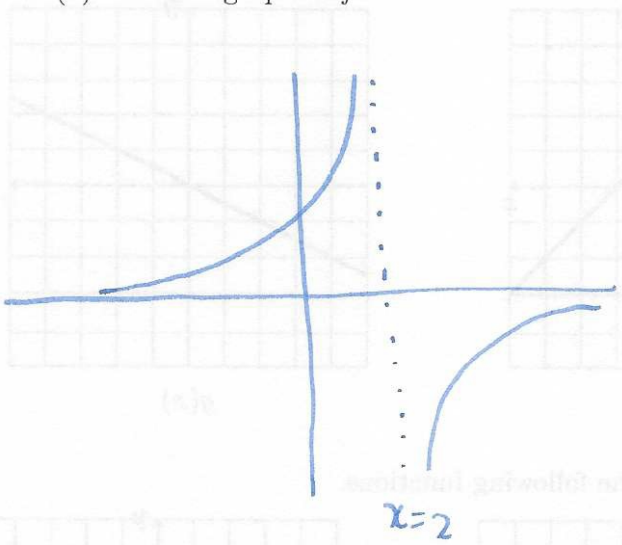
$f(g(4)) = f(2) = 0$

(c) Does g have an inverse? If so, find $g^{-1}(-1)$.

Yes. $g^{-1}(-1) = -2$

(5) (10 points) Let $f(x) = \dfrac{1}{2-x}$.

(a) Sketch a graph of f.



(b) Does f have an inverse? If so, find a formula for the inverse.

Yes.

$$y = \frac{1}{2-x}$$

$$y(2-x) = 1$$

$$2y - xy = 1$$

$$2y - 1 = xy$$

$$x = \frac{2y-1}{y} = 2 - \frac{1}{y}$$

$$\boxed{f^{-1}(x) = 2 - \frac{1}{x}}$$

(6) (10 points) Use the method of completing the square to find the largest value of the function $2 + x - 2x^2$.

$-2x^2 + x + 2$

$-2\left(x^2 - \frac{1}{2}x - 1\right)$

$-2\left(\left(x - \frac{1}{4}\right)^2 - \frac{1}{16} - 1\right) = -2\left(\left(x - \frac{1}{4}\right)^2 - \frac{17}{16}\right)$

$-2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - 1\right) = -2\left(x - \frac{1}{4}\right)^2 + \frac{34}{16}$

Largest value of function: $\frac{34}{16} = \frac{17}{8}$

(7) (10 points) Find the values of x for which $x^2 + x < 6$.

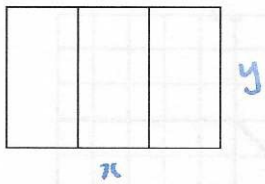
$x^2 + x - 6 < 0$

$(x-2)(x+3) < 0$

	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
$(x-2)$	$-$	$-$	$+$
$(x+3)$	$-$	$+$	$+$
$(x-2)(x+3)$	$+$	$-$	$+$

answer: $(-3, 2)$

(8) (10 points) A farmer wishes to create three adjacent rectangular fields, as drawn below.



If the farmer has 400ft of fencing, what is the maximum area of the fields?

area $A = xy$

length $400 = 2x + 4y$

$200 = x + 2y \qquad x = 200 - 2y$

area $A = (200-2y)y = 200y - 2y^2$

$= -2y^2 + 200y$

$= -2(y^2 - 100y)$

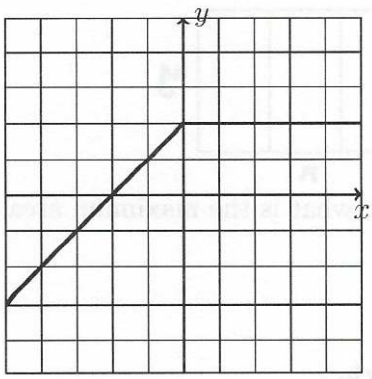
$= -2((y-50)^2 - 2500) = -2(y-50)^2 + 5000$

$= -2(y^2 - 100y + 2500 - 2500)$

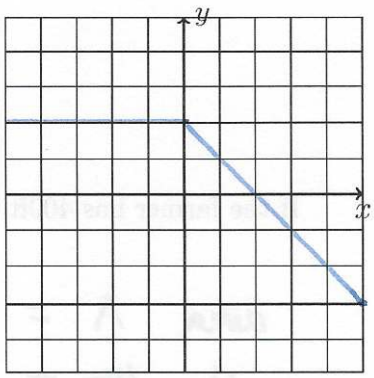
Maximum area: $5000\ ft^2$

(9) (10 points) The graph of the function $f(x)$ is shown below.

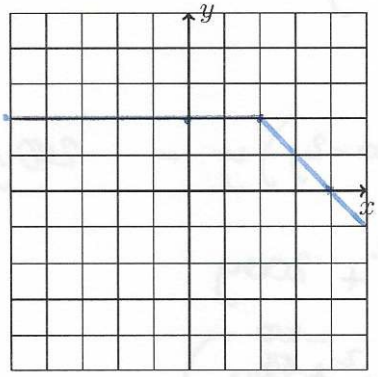
(a) Sketch the graphs of the other functions. Hint: do them in order.



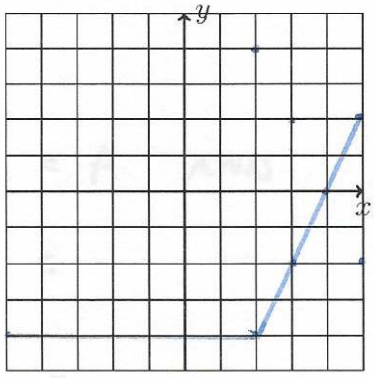
$f(x)$



$f(-x)$



$f(-x+2)$



$-2f(-x+2)$

(b) What is the average value of $f(x)$ between $x = -1$ and $x = 1$?

average value $= \dfrac{f(1)-f(-1)}{1-(-1)} = \dfrac{2-1}{2} = \dfrac{1}{2}$

(10) (10 points) Let $f(x) = 2x^2 - x - 3$.

(a) What value of x gives the minimum value of the function?

$2\left(x^2 - \frac{1}{2}x\right) - 3$

$2\left(\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right) - 3 = 2\left(x - \frac{1}{4}\right)^2 - \frac{1}{8} - 3$

$2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right) - 3 = 2\left(x - \frac{1}{4}\right)^2 - \frac{25}{8}$

minimum value occurs at $x = \frac{1}{4}$

(b) What is the minimum value of the function?

minimum value of function is $-\frac{25}{8}$