MTH 130 Precalculus, Classwork 17 **Fall 2014**

(1) A ferris wheel has a radius of 11m and the bottom of the wheel passes 1m above the ground. If the ferris wheel makes one complete revolution every 20s, find an equation that gives the height above the ground of a person on the ferris wheel as a function of time, assuming at $t = 0$ person starts at bottom (i.e. height 1m). (Hint: draw pictures).

(2) Verify:

(a) $\dfrac{1}{1-\sin^2 y} = 1 + \tan^2(y)$

(b) $(1-\cos^2 x)(1+\cot^2 x) = 1$

(c) $\dfrac{\sin(x)+\cos(x)}{\sec(x)+\csc(x)} = \sin(x)\cos(x)$

(3) Rewrite:

(a) $\cos^4 x$ as powers of sine.

(b) $\cos^4 x$ as first powers of cosine.

(c) $\sin 2x - \cos 2x$ in terms of sine.

(4) Find the exact value of:

(a) $\sin(-\frac{7\pi}{12})$.

(b) $\cos(\frac{13\pi}{15})\cos(-\frac{\pi}{5}) - \sin(\frac{13\pi}{15})\sin(-\frac{\pi}{5})$.

(5) Verify:

(a) $\sin(x+y) - \sin(x-y) = 2\cos x \sin y$.

(b) $\cos(x+y) + \cos(x-y) = 2\cos x\cos y$.

(6) Write in terms of sine only:

(a) $-\sqrt{3}\sin x + \cos x$.

(b) $\sin 2x - \cos 2x$.

3a) $\cos^4 x = (1-\sin^2 x)^2$

$= 1 - 2\sin^2 x + \sin^4 x$

b) $\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$

$\cos^4 x = \frac{1}{4}(\cos 2x + 1)^2$

$= \frac{1}{4}(\cos^2 2x + 2\cos 2x + 1)$

$\cos^2 2x = \frac{1}{2}\cos 4x + \frac{1}{2}$.

$\Rightarrow \cos^4 x = \frac{1}{8}\cos 4x + \frac{1}{8} + \frac{1}{2}\cos 2x + \frac{1}{4}$.

$= \frac{1}{8}\cos 4x + \frac{1}{2}\cos 2x + \frac{3}{8}$

c) $= 2\sin x\cos x - \cos^2 x + \sin^2 x$

$= 2\sin x(\pm\sqrt{1-\sin^2 x}) - 1 + 2\sin^2 x$

(or just $\sin 2x - 1 + 2\sin^2 x$).

(or see Q6 b).

Q4 a) $\sin\left(-\frac{7\pi}{12}\right) = -\sin\left(\frac{7\pi}{12}\right) = -\sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= -\sin\frac{\pi}{4}\cos\frac{\pi}{3} - \cos\frac{\pi}{4}\sin\frac{\pi}{3}.$

$= -\frac{\sqrt{2}}{2}\,\frac{1}{2} - \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2}$

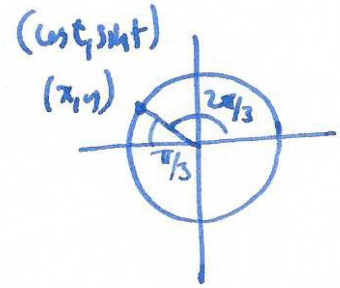
$= -\frac{1}{4}\left(\sqrt{2}+\sqrt{6}\right)$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos x$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

b) $\cos\left(\frac{13\pi}{5}\right)\cos\left(-\frac{\pi}{5}\right) - \sin\left(\frac{13\pi}{15}\right)\sin\left(-\frac{\pi}{5}\right) = \cos\left(\frac{13\pi}{15} - -\frac{\pi}{5}\right)$

$= \cos\left(\frac{10\pi}{15}\right) = \cos\left(\frac{2\pi}{3}\right)$

$= -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}.$



Q5 a) $\sin(x+y) = \sin x\cos y + \cos x\sin y$

$\sin(x-y) = \sin x\cos y - \cos x\sin y$

$\sin(x+y) - \sin(x-y) = 2\cos x\sin y$

b) $\cos(x+y) = \cos x\cos y - \sin x\sin y$

$\cos(x-y) = \cos x\cos y + \sin x\sin y$

$\cos(x+y) + \cos(x-y) = 2\cos x\cos y.$

Q6 a) $-\sqrt{3}\sin x + \cos x$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B.$$

$$A\sin(x+B) = A\sin x\cos B + A\cos x\sin B$$

want: $\left.\begin{aligned} A\cos B &= -\sqrt{3} \\ A\sin B &= 1 \end{aligned}\right\}$ $\tan B = -\frac{1}{\sqrt{3}}.$ $B = -\frac{\pi}{6}$

$A^2\cos^2 B + A^2\sin^2 B = 3+1$

$A^2 = 4$ $A = 2$

$$-\sqrt{3}\sin x + \cos x = 2\sin\left(x - \frac{\pi}{6}\right).$$

b) $\sin(A+B) = \sin A\cos B + \cos A\sin B.$

$$A\sin(2x+B) = A\sin 2x\cos B + A\cos 2x\sin B$$

want: $\left.\begin{aligned} A\cos B &= 1 \\ A\sin B &= -1 \end{aligned}\right\}$ $\tan B = -1 \Rightarrow B = -\frac{\pi}{4}$

$A^2\cos^2 B + A^2\sin^2 B = 1+1$

$A^2 = 2$ $A = \sqrt{2}.$

$$\sin 2x - \cos 2x = \sqrt{2}\sin\left(2x - \frac{\pi}{4}\right).$$