MTH 130 Precalculus, Classwork 5 **Fall 2014**

(1) (a) Complete the square for: $3 - 2x - 4x^2$.

$-4x^2-2x+3 = -4\left(x^2+\frac{1}{2}x\right)+3 = -4\left((x+\frac{1}{4})^2 - \frac{1}{16}\right)+3 = -4\left(x+\frac{1}{4}\right)^2+\frac{1}{4}+3$

$-4\left(x^2+\frac{1}{2}x+\frac{1}{16}-\frac{1}{16}\right)+3 \qquad -4\left(x+\frac{1}{4}\right)^2+\frac{13}{4}$

(b) What is the maximum value of $\sin\theta - \sin^2\theta$?

$-(\sin^2\theta-\sin\theta) = -\left(\left(\sin\theta-\frac{1}{2}\right)^2-\frac{1}{4}\right) = -\left(\sin\theta-\frac{1}{2}\right)^2+\frac{1}{4}$ max value $= 1/4$.

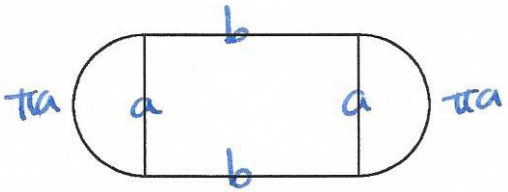
$\sin^2\theta-\sin\theta+\frac{1}{4}-\frac{1}{4}$

(c) What value of t ~~minimizes~~ maximizes $1 + 4t - 2t^2$?

$-2t^2+4t+1 = -2\left(t^2-2t\right)+1 = -2\left((t-1)^2-1\right)+1 = -2(t-1)^2+3$

$t^2-2t+1-1$

(2) I wish to build a running track in the shape of a rectangle with two semi-circles added to each end:



(a) If the total perimeter should be 800m, what dimensions give the maximum area?

Area $A = ab + \pi\left(\frac{a}{2}\right)^2 = ab + \frac{\pi}{4}a^2$

Perimeter $= 2b + 2\pi a = 800 \Rightarrow b = 400 - \pi a$

$A = a(400-\pi a)+\frac{\pi}{4}a^2$

$= 400a - a^2\left(\pi-\frac{\pi}{4}\right)$.

$= 400a - \frac{3\pi}{4}a^2$.

(b) If the total area should be 500m^2, what is the shortest possible length of the track?

complete the square for $-\frac{3\pi}{4}a^2+400a$

$-\frac{3\pi}{4}\left(a^2-\frac{1600}{3\pi}a\right)$

$-\frac{3\pi}{4}\left(\left(a-\frac{800}{3\pi}\right)^2-\left(\frac{800}{3\pi}\right)^2\right)$

$-\frac{3\pi}{4}\left(a^2-\frac{1600}{3\pi}a+\left(\frac{800}{3\pi}\right)^2-\left(\frac{800}{3\pi}\right)^2\right.$

so max area is $+\frac{3\pi}{4}\cdot\left(\frac{800}{3\pi}\right)^2$.