Math 130 Precalculus Fall14, FINAL b

Name: Solutions

- No notes
- Cellphones must be switched OFF.
- You MUST EXPLAIN your answers and MUST show all your work

1	12	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	8	
12	8	
	100	

(1) (**12 points**) Consider the function

$$f(x) = \frac{x-4}{x-2}$$

(a) (2 points) Find the maximal domain for this function.

$x \neq 2$ $\quad (-\infty, 2) \cup (2, \infty)$

(b) (2 points) Find the x- and y-intercepts.

y-intercept : $f(0) = 2$

x-intercept : solve $f(x) = 0$ $\quad x = 4$

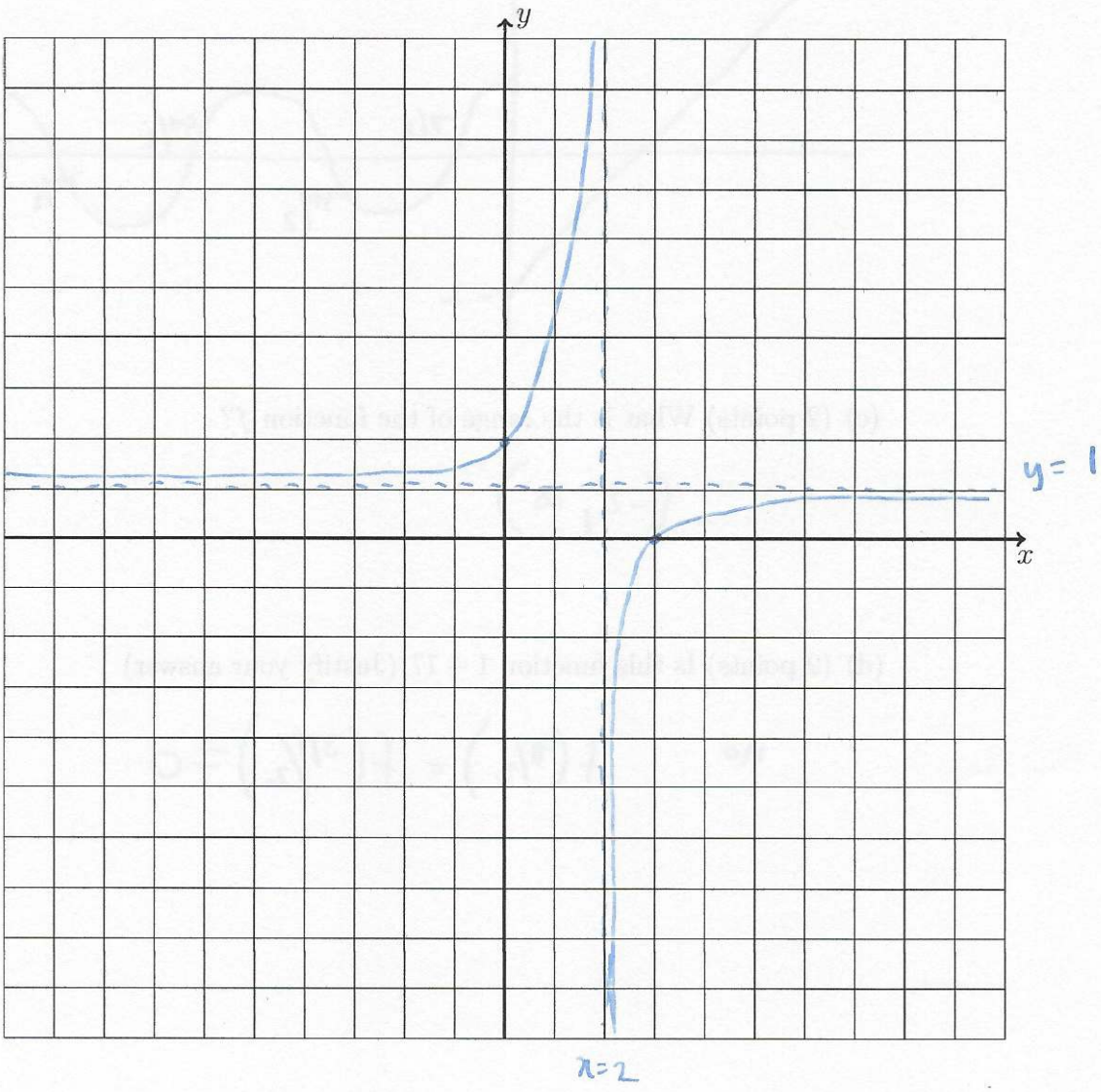
(c) (2 points) Examine how the function behaves when $x \to \pm\infty$

$\frac{x-4}{x-2} \sim \frac{x}{x} = 1$.

(d) (2 points) Find equations of all the vertical asymptotes (if any).

$x = 2$

(e) (4 points) Based on all this information, sketch the graph of this function and find its range.



(2) (**8 points**) Consider the function

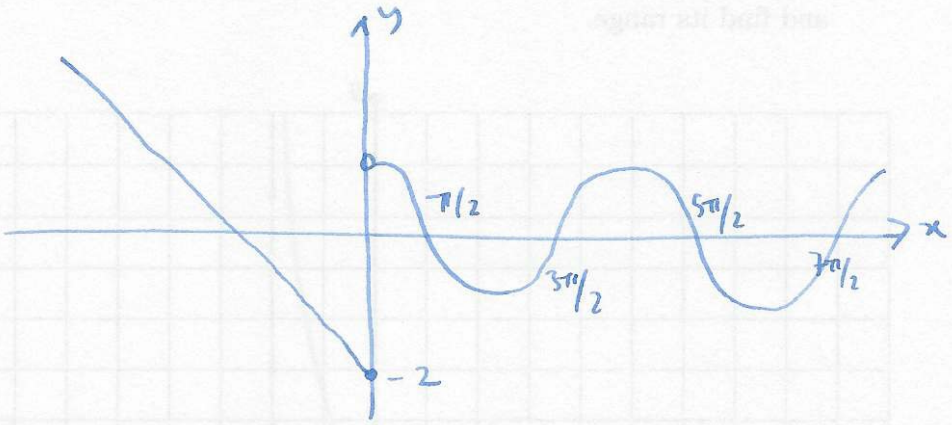
$$f(x) = \begin{cases} \cos x & x > 0 \\ -x - 1 & x \leq 0 \end{cases}$$

(a) (2 points) Find the values of $f(0)$, $f(-\pi)$ and $f(\pi)$.

$f(0) = -1$ $\quad f(-\pi) = \pi - 1$

$f(\pi) = \cos(\pi) = -1$

(b) (2 points) Sketch the graph of the function f, indicating the x- and y-intercepts.



(c) (2 points) What is the range of the function f?

$[-2, \infty)$

(d) (2 points) Is this function $1-1$? (Justify your answer)

no. $f(\pi/2) = f(3\pi/2) = 0$

(3) (**8 points**) A quadratic function is given $f(x) = 2x^2 - 3x + 1$.

(a) (2 points) Express the quadratic function in the standard form

$$f(x) = a(x-h)^2 + k$$

$2\left(x^2 - \frac{3}{2}x + \frac{1}{2}\right)$

$2\left(\left(x - \frac{3}{4}\right)^2 - \frac{9}{16} + \frac{1}{2}\right) = 2\left(x - \frac{3}{4}\right)^2 - \frac{9}{8} + 1$

$2\left(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16} + \frac{1}{2}\right)$ $\quad = \quad 2\left(x - \frac{3}{4}\right)^2 - \frac{1}{8}$

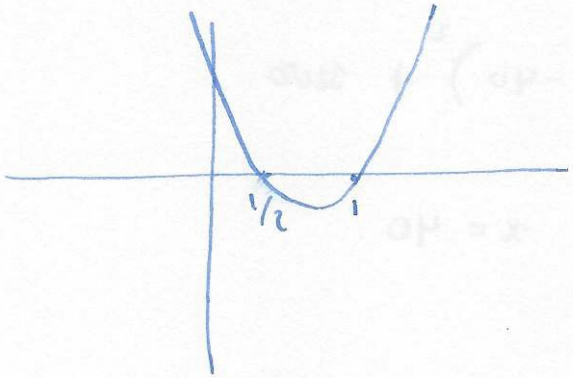
(b) (2 points) Find the coordinates of the minimum point on the graph.

$\left(\frac{3}{4}, -\frac{1}{8}\right)$

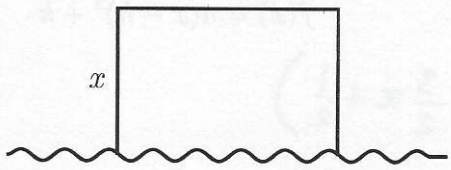
(c) (2 points) Find the x-intercepts

$(2x - 1)(x - 1)$ $\quad x = \frac{1}{2}, \; x = 1.$

(d) (2 pt) Sketch the graph of f.



(4) (**8 points**) A farmer has 160 feet of fencing, and wishes to build three fences to create a rectangular fields with one boundary by a river, as illustrated below.



(a) (2 points) Let x be the length of the fence perpendicular to the river. Write down a formula for the total area $A(x)$ of the field in terms of x.

$A(x) = x(160 - 2x)$

$= -2x^2 + 160x$

(b) (6 points) How should the farmer choose the value of x in order to maximize the total area $A(x)$?

$A(x) = -2(x^2 - 80x)$

$= -2((x-40)^2 - 1600)$

$-2((x^2 - 80x + 1600) - 1600)$

$= -2(x-40)^2 + 3200$

choose $x = 40$.

(5) (**8 points**) Consider the polynomial $P(x) = x^6 - 2x^4 - 8x^2$.

(a) (6 points) Find all zeros (real and complex) of the polynomial $P(x)$.

$x^2(x^4 - 2x^2 - 8)$

$x^2(x^2-4)(x^2+2)$

$x=0 \quad x=\pm 2 \quad x=\pm\sqrt{2}i$

(b) (2 points) Write $P(x)$ as a product of linear polynomials.

$x^2(x-2)(x+2)(x+\sqrt{2}i)(x-\sqrt{2}i)$.

(6) (**8 points**) You put \$600 in a bank account with 12% interest per year.

(a) (2 points) If the interest is compounded **monthly**, how much will you have after 1 month ?

$$A\left(\tfrac{1}{12}\right) = 600\left(1+\frac{0.12}{12}\right)^{12\cdot\frac{1}{12}} = 606$$

(b) (2 points) If the interest is compounded **continuously**, how much will you have after 12 months ?

$$A(1) = 600\, e^{0.12\times 1} \approx 676.50$$

(c) (4 points) If the interest is compounded **continuously**, how long will it take for you to have \$1000 ?

$$600\, e^{0.12t} = 1000$$

$$e^{0.12t} = \frac{1000}{600} = \frac{5}{3}$$

$$t = \frac{\ln(5/3)}{0.12} \approx 4.26 \text{ years}$$

(7) (**8 points**) Solve the following equations

(a) (4 points) $e^{2x} - e^x - 6 = 0$

$$(e^x - 3)(e^x + 2) = 0$$

$e^x = 3$ $\qquad$ $e^x = -2$

$\Rightarrow$ $x = \ln(3)$ $\qquad$ no solutions.

(b) (4 points) $\log_4(x+5) - \log_4(x-1) = 2$

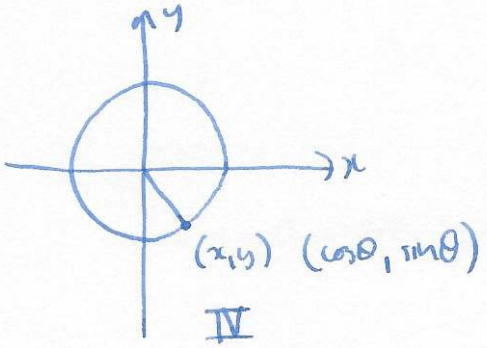
$$\frac{x+5}{x-1} = 4^2 = 16$$

$$x + 5 = 16x - 16$$

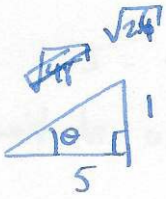
$$21 = 15x$$

$$x = \frac{21}{15} = \frac{7}{5}$$

(8) (**8 points**) Let θ be an angle with $\cot\theta = -5$, and with the terminal point of θ in the fourth quadrant. Calculate exact values of all six trigonometric functions of θ.



$\cot\theta = -5$ $\quad \tan\theta = -\frac{1}{5}$



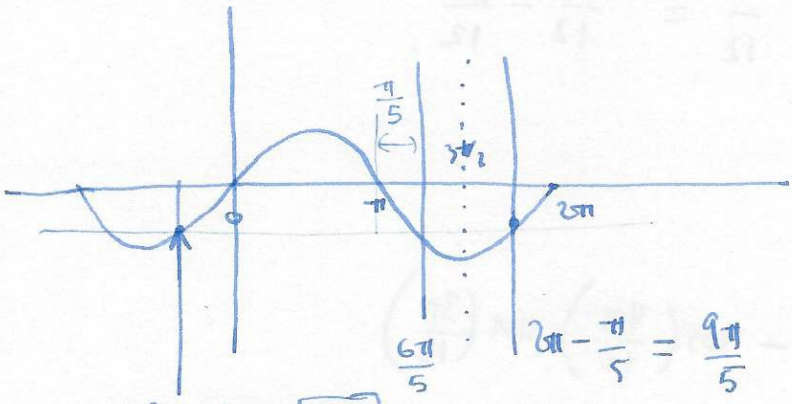
$\sin\theta = -\frac{1}{\sqrt{26}}$ $\quad \csc\theta = \sqrt{26}$

$\cos\theta = \frac{5}{\sqrt{26}}$ $\quad \sec\theta = \frac{\sqrt{26}}{5}$

(9) (**8 points**) Find the exact values of

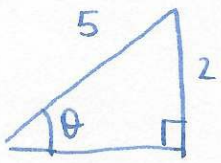
(a) $\sin^{-1}(\sin(16\pi/5))$

$\frac{16\pi}{5} = 3\pi + \frac{\pi}{5}$ so $\sin\left(\frac{16\pi}{5}\right) = \sin\left(\frac{6\pi}{5}\right)$.



$\frac{9\pi}{5} - \frac{10\pi}{5} = \boxed{-\frac{\pi}{5}}$

(b) $\tan(\sin^{-1}(2/5))$



$\sqrt{25-4} = \sqrt{21}$

$\tan\theta = \frac{2}{\sqrt{21}}$

(10) (**8 points**) Find the exact value of $\sin(\pi/12)$. You may use the facts that

$$\sin(\pi/4) = \frac{\sqrt{2}}{2} \quad \text{and} \quad \sin(\pi/3) = \frac{\sqrt{3}}{2}$$

$\frac{\pi}{4} = \frac{3\pi}{12}$ $\frac{\pi}{3} = \frac{4\pi}{12}$ $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12}$.

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$$

$$= \sin\left(\frac{4\pi}{12}\right)\cos\left(\frac{3\pi}{12}\right) - \cos\left(\frac{4\pi}{12}\right)\sin\left(\frac{3\pi}{12}\right)$$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\frac{\sqrt{3}}{2}\quad\frac{\sqrt{2}}{2} \quad - \quad \frac{1}{2}\quad\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

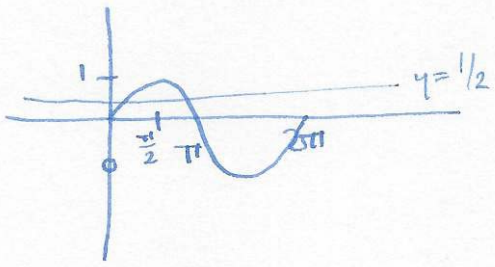
(11) (**8 points**) Find **all** solutions to the equation

$$2(\sin x)^2 - 3\sin x + 1 = 0.$$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n$



$x = \frac{\pi}{6}, \frac{5\pi}{6} + 2\pi n$

(12) (**8 points**) Prove the following identity

$$\sec\theta \sin^2\theta = \sec\theta - \cos\theta.$$

$$\frac{1}{\cos\theta}(1-\cos^2\theta) = \frac{1}{\cos\theta} - \cos\theta$$

$$= \sec\theta - \cos\theta$$

END OF EXAMINATION