# §5.6 substitution / change of variable

"reverse chain rule for integration"

recall: chain rule: $\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$

substitution / change of variable

$\int f(u)\,du \quad u(x)$ : $\boxed{\int_a^b f(u)\,du = \int_{u^{-1}(a)}^{u^{-1}(b)} f(u(x))\,\frac{du}{dx}\,dx}$

remember: three things to change: limits, function, differential

mnemonic: $du = \frac{du}{dx}\,dx$

why does this work?

$\int f(u)\,du$, $u(x)$ set $F(u) = \int f(u)\,du$, so $F'(u) = f(u)$

$\int f(u)\,du = F(u) \underset{\text{wrt } x}{\overset{\text{diff}}{\rightsquigarrow}} \frac{d}{dx}(F(u)) = F'(u(x))\cdot u'(x)$

$\underset{\text{wrt } x}{\overset{\text{int}}{\rightsquigarrow}} \int f(u(x))\,u'(x)\,dx = \int f(u(x))\frac{du}{dx}\,dx \quad \square$

Examples

$\int_{x=0}^{x=1} e^{-7x}\,dx$ set $u = -7x$

$\frac{du}{dx} = -7$

useful fact: $\boxed{\frac{dx}{du} = \frac{1}{\frac{du}{dx}}}$

$\int_{u=0}^{u=-7} e^u \frac{dx}{du}\,du = \int_0^{-7} e^u\cdot -\frac{1}{7}\,du = -\frac{1}{7}\left[e^u\right]_0^{-7} = -\frac{1}{7}(e^{-7}-1)$