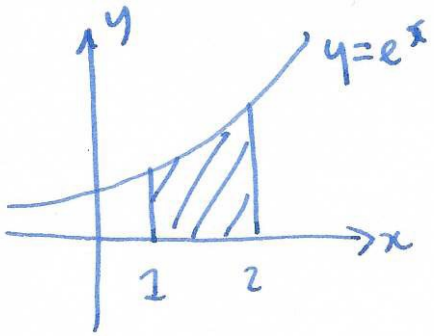
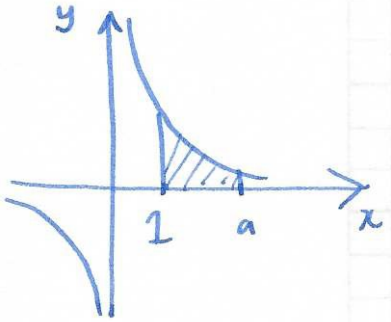


$$\int_1^2 e^x\,dx = \left[e^x\right]_1^2 = e^2 - e$$



$$\int_1^a \frac{1}{x}\,dx = \left[\ln|x|\right]_1^a = \ln(a) - \ln(1) = \ln(a).$$

observations

① choice of anti-derivative doesn't matter: let $F(x)$ and $F(x)+c$ be anti-derivatives for $f(x)$. Then $\int_a^b f(x)\,dx = F(b) - F(a)$

$$= F(b) + c - (F(a) + c) = F(b) - F(a)$$

② $\int_a^t f(x)\,dx$ is a function of t! x is called a dummy variable.

i.e. $\int_a^t f(x)\,dx = \int_a^t f(y)\,dy$, if you want a function of x use $\int_a^x f(t)\,dt$.

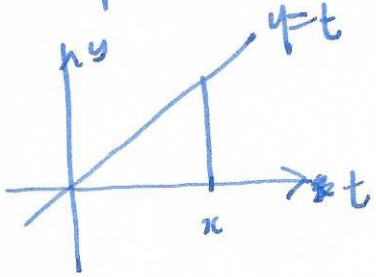
§5.4 Fundamental theorem of calculus II

Thm (FTC ②) Let $f(x)$ be continuous on $[a,b]$, then

$A(x) = \int_a^x f(t)\,dt$ is an anti-derivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx}$

i.e. $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. Furthermore $A(a) = 0$.

Examples



$$\int_0^x t\,dt = \left[\tfrac{1}{2}t^2\right]_0^x = \tfrac{1}{2}x^2 - 0 = \tfrac{1}{2}x^2$$

Example $\int_0^x e^{-t^2}\,dt$ ← a function with derivative e^{-x^2}.

Example what about $\int_0^{x^2} \sin(t)\,dt$ ← this is a function of a function!

set $A(x) = \int_0^x \sin(t)\,dt$, then $G(x) = \int_0^{x^2} \sin(t)\,dt = A(x^2)$

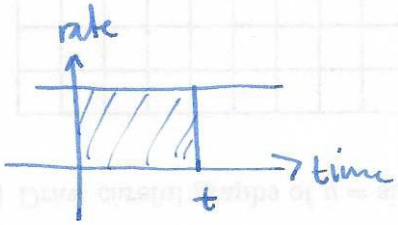
use chain rule! $\frac{d}{dx}(G(x)) = \frac{d}{dx}(A(x^2)) = A'(x^2)\cdot\frac{d}{dx}(x^2)$

$= \sin(x^2)\cdot 2x$.

§5.5 Net change / applications of integrals

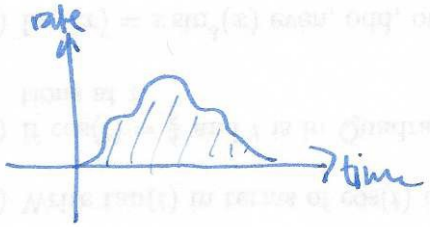
Example you pour water into a bucket at rate $r(t)$. How much water is in the bucket?

constant rate:



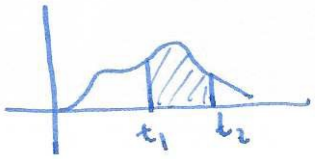
amount = rate × time

varying rate:



amount = area under the curve $= \int_0^t r(x)\,dx$

Q: how much did the amount of water change between t_1 and t_2?



net change $= \int_{t_1}^{t_2} r(x)\,dx$.