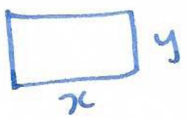
# §4.6 Optimization

**Example** A piece of wire of length $L$ is bent into a rectangle. What's the largest possible area?

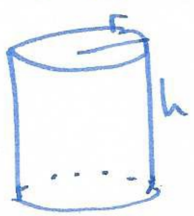


$$\left.\begin{array}{l} \text{area } A = xy \\ \text{perimeter } L = 2x + 2y \end{array}\right\} \quad y = \frac{L}{2} - x \qquad A = x\left(\frac{L}{2} - x\right) = \frac{L}{2}x - x^2$$

$A'(x) = \frac{L}{2} - 2x$ critical point $x = \frac{L}{4}$ (max)

so max area of $\frac{L^2}{16}$ occurs when $x = y = \frac{L}{4}$ (square)

**Example** What shape of cylindrical can minimizes surface area, if you want total volume to be $1\ ft^3$?



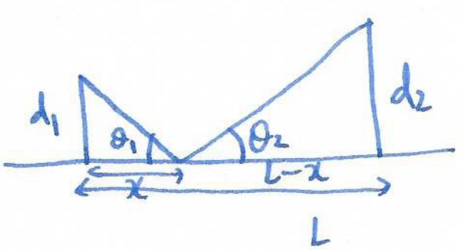
$$V = \pi r^2 h = 1 \longleftrightarrow h = \frac{1}{\pi r^2}$$

$$A = 2\pi r^2 + 2\pi r h$$

$$A = 2\pi r^2 + \frac{2\pi r}{\pi r^2} = 2\pi r^2 + \frac{2}{r}$$

$A'(r) = 4\pi r - \frac{2}{r^2}$ solve $A'(r) = 0$ $\quad r^3 = \frac{1}{2\pi}$.

**Example** a ball bounces off a wall. Show that the path which minimizes distance has equal angle of incidence and reflection



$$f(x) = \text{distance} = \sqrt{d_1^2 + x^2} + \sqrt{d_2^2 + (L-x)^2}$$

$$f'(x) = \frac{1}{2}\left(d_1^2 + x^2\right)^{-1/2} \cdot 2x + \frac{1}{2}\left(d_2^2 + (L-x)^2\right) \cdot 2(L-x) \cdot -1$$

so $f'(x) = 0$: $\quad \dfrac{x}{\sqrt{d_1^2 + x^2}} = \dfrac{L-x}{\sqrt{d_2^2 + (L-x)^2}} \longleftrightarrow \cos\theta_1 = \cos\theta_2 \longleftrightarrow \theta_1 = \theta_2$.

## §4.9 Antiderivatives

**Defn** A function $F(x)$ is an <u>anti-derivative</u> of $f(x)$ if $F'(x) = f(x)$

**Example** $f(x) = x^2$, then $F(x) = \frac{1}{3}x^3$ is an anti-derivative.

check: $\frac{d}{dx}\left(\frac{1}{3}x^3\right) = \frac{1}{3}3x^2 = x^2 = f(x)$ ✓.

**Note** $F(x) = \frac{1}{3}x^3 + 4$ is also an antiderivative: $\frac{d}{dx}\left(\frac{1}{3}x^3+4\right) = x^2$.

**General anti-derivative**

**Thm** Let $F(x)$ be an anti-derivative for $f(x)$, then any other anti-derivative has the form $F(x)+c$ for some $c \in \mathbb{R}$.

**Proof** suppose $F(x)$ and $G(x)$ are anti-derivatives for $f(x)$.

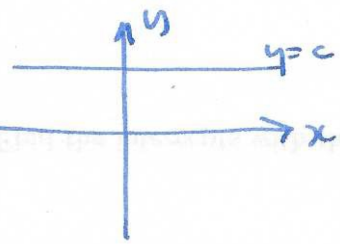
i.e. $F'(x) = f(x)$ and $G'(x) = f(x)$

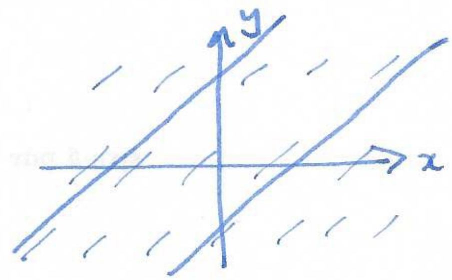
consider $G(x) - F(x)$, which has derivative $(G(x)-F(x))' = f(x) - f(x) = 0$

so $G(x) - F(x) = c$ constant function. $\square$.

**Picture**: $f(x)$ gives the slope function for $F(x)$.

**Examples** $f(x) = c$

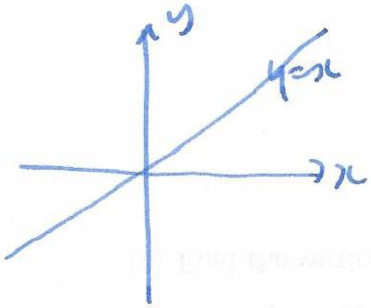


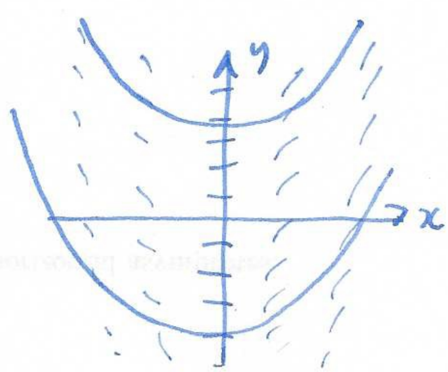




slope $F'(x) = c$ everywhere

$F(x) = cx + d$

$f(x) = x$




$F(x) = \frac{1}{2}x^2 + c$