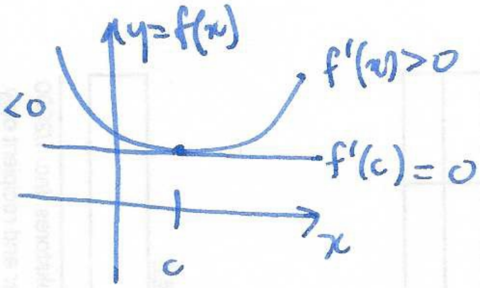
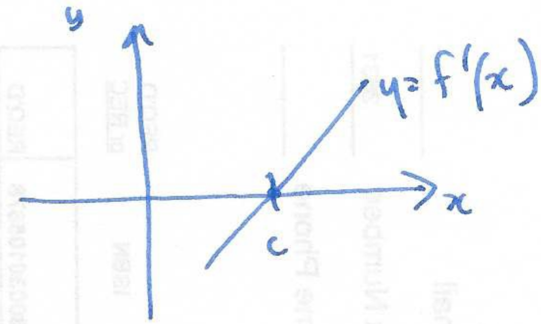
<u>local min</u>:





$f'(x)$ goes from negative to positive $\Rightarrow$ local min.

<u>Th$^{\underline{m}}$</u> <u>First derivative test</u>

If $f(x)$ is differentiable and $f'(c)=0$ (e.g. c is a critical point)

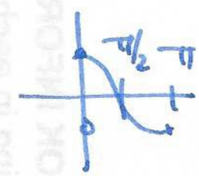
then if $f'(x)$ changes from $+$ve to $-$ve at $c \Rightarrow c$ local max

$-$ve to $+$ve $\Rightarrow c$ local min

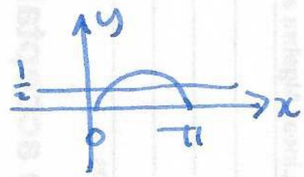
<u>Example</u> classify critical points of $f(x) = \cos^2 x + \sin x$ on $[0, \pi]$

find critical points : $f'(x) = -2\cos(x)\sin(x) + \cos(x)$

solve : $f'(x)=0$ $\qquad \cos(x)\left[1-2\sin(x)\right]=0$ $\qquad \cos(x)=0$



$1-2\sin x = 0 \Leftrightarrow \sin x = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

so critical points are $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

find sign of $f'(x)$ between them :

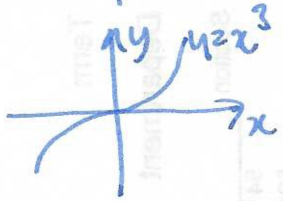
		$\pi/6$		$\pi/2$		$5\pi/6$	
$\cos(x)$	$+$		$+$		$-$		$-$
$1-2\sin(x)$	$+$		$-$		$-$		$+$
$f'(x)$	$+$		$-$		$+$		$-$

$\Rightarrow$ local max $\frac{\pi}{6}, \frac{5\pi}{6}$

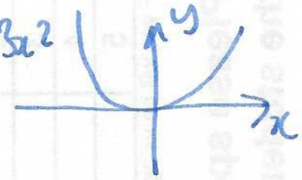
local min $\pi/2$.

Example critical point not a local max or min

$f(x) = x^3$



$f'(x) = 3x^2$



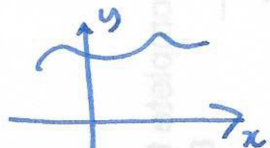
$f'(c)=0$

$f'(x)$ + 0 +

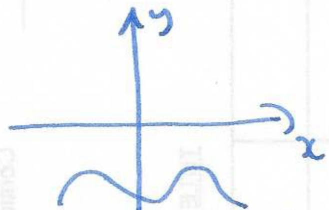
not local max or min.

§4.4 Second derivative test

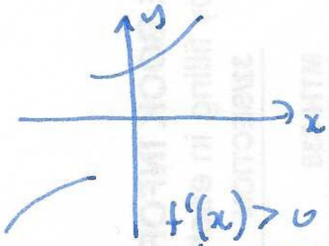
recall



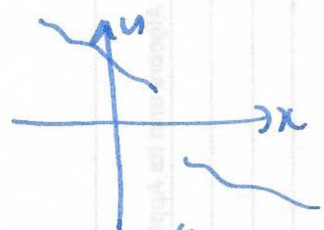
$f(x)>0$ positive values



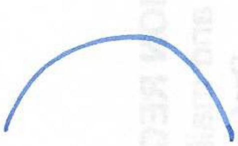
$f(x)<0$ negative values



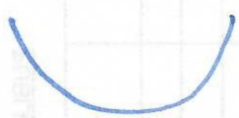
$f'(x)>0$ increasing



$f'(x)<0$ decreasing



concave down



concave up

mnemonic

down up

Q: how do the slopes change?

concave down

$\Leftrightarrow$ slopes decreasing

$\Leftrightarrow$ $f''(x)<0$

concave up

$\Leftrightarrow$ slopes increasing

$\Leftrightarrow$ $f''(x)>0$

Defⁿ Let $f(x)$ be differentiable on an interval (a,b). Then

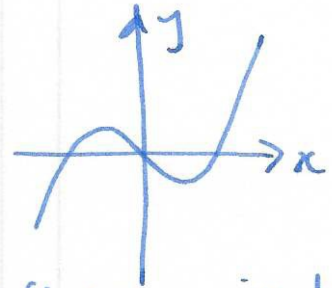
$f(x)$ is concave up $\iff f''(x) > 0$

$f(x)$ is concave down $\iff f''(x) < 0$

Defⁿ A point of inflection is where the graph changes from concave up to concave down (or vice versa).

Note: x point of inflection $\underset{\not\Leftarrow}{\Rightarrow} f''(x) = 0$

Example $f(x) = x^3 - x = x(x^2-1) = x(x-1)(x+1)$



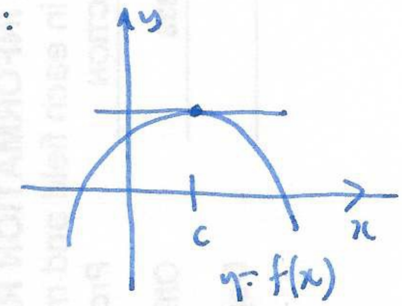
$f'(x) = 3x^2 - 1$

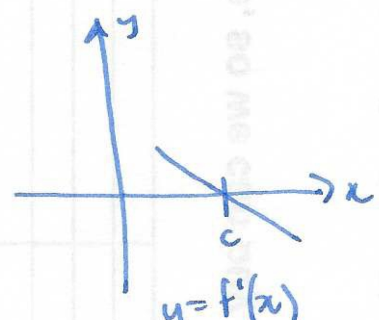
$f''(x) = 6x$

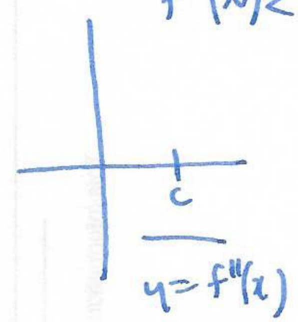
$\left.\begin{array}{l} f''(x) > 0 \text{ for } x > 0 \\ f''(x) < 0 \text{ for } x < 0 \end{array}\right\}$ inflection point at $x = 0$.

Second derivative test

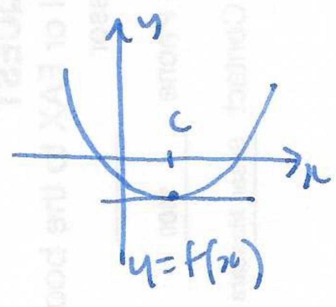
local max:

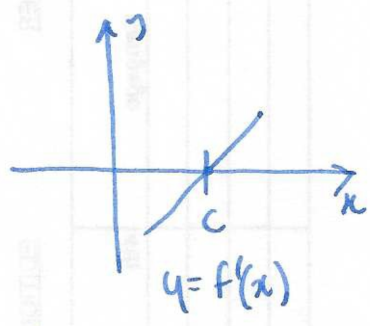


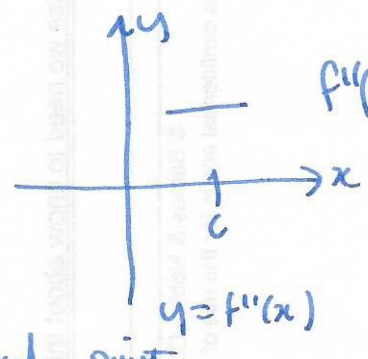




local min:







Thⁿ Suppose $f(x)$ is differentiable and c is a critical point

If $f''(c) > 0 \Rightarrow c$ is a local max

$f''(c) < 0 \Rightarrow c$ is a local min

$f''(c) = 0 \Rightarrow$ NO INFORMATION! (may be local max/min/neither)

Example: $f(x) = x^5 - 5x^4$