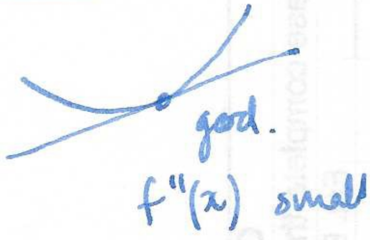
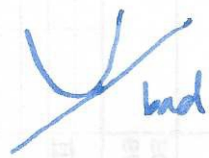
Q: is this good or bad?

absolute error = 11

percentage error = $\left|\dfrac{\text{error}}{\text{actual value}}\right| \times 100 = \left(\dfrac{11}{\pi 18^2/4}\right) \times 100$.

observation: when is the linear approximation a good approximation?

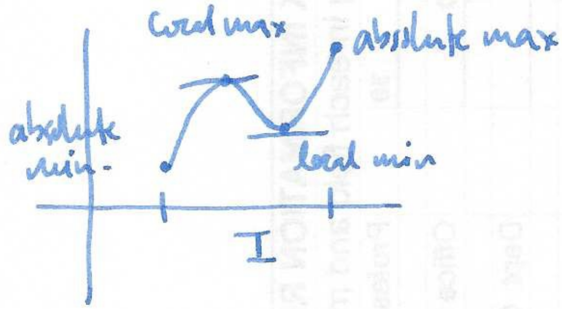




$f''(x)$ large.

§4.2 Extreme values

Suppose $f(x)$ is defined on an interval I



Def^{n} $f(a)$ is the absolute max if

$f(a) \geq f(x)$ for all $x \in I$

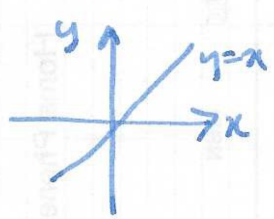
$f(a)$ is the absolute min if

$f(a) \leq f(x)$ for all $x \in I$.

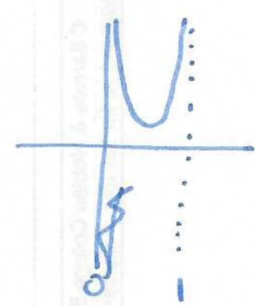
note: if some one asks for absolute max/min usually want value of function, not x-value giving that.

warning: some functions do not have a max or min

Example $f: \mathbb{R} \to \mathbb{R}$
$x \mapsto x$



$f: (0,1) \to \mathbb{R}$
$x \mapsto \dfrac{1}{x(1-x)}$



Th^{m} If $f(x)$ is continuous on a closed bounded interval then $f(x)$ has both an absolute max and an absolute min

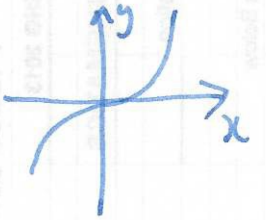
Def^{n} $f(x)$ has a local max at $x = c$ if $f(c)$ is the max value for f on some small interval containing c. Similarly for local min.

Defn we say that c is a critical point if $f'(c)=0$ (or $f'(c)$ undefined)

Thm If c is a ~~critical point~~ local max or min, then $f'(c)=0$, i.e. a critical point.

Warning $f'(c)=0 \not\Rightarrow$ local max or min

Example $y=x^3$



$f'(x)=3x^2 \quad f'(0)=0$

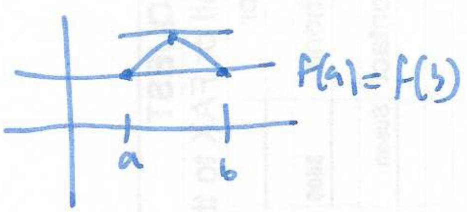
but $x=0$ not local max or min.

How to find absolute max or min of a ~~cts~~ diff'ble function on a closed interval $[a,b]$:
1. find critical points, and evaluate function there.
2. check endpoints.

Example find absolute max and min of
$$2x^3-15x^2+24x+7 \text{ on } [0,3]$$
$$x^2-8 \text{ on } [1,4]$$
$$\cos(x).\sin(x) \text{ on } [0,\pi]$$

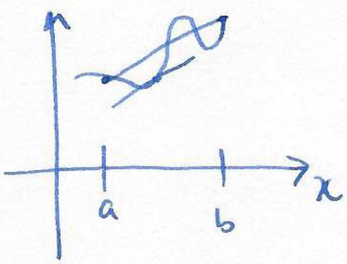
Thm (Rolle's Thm) Suppose $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b). If $f(a)=f(b)$, then there is a $c\in[a,b]$ s.t. $f'(c)=0$.

Proof



- if there is a local max or min at c then $f'(c)=0$
- if no local max or min then $f(x)=\text{const}$ so $f'(c)=0$ for all c.

§4.3 First derivative test



Thm (Mean Value Theorem) (MVT)

Suppose f is continuous on $[a,b]$ and differentiable on (a,b), then there is a $c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

i.e. there is a point where the slope is equal to the average rate of change.

Corollary If $f(x)$ is differentiable, and $f'(x)=0$ then $f(x)=c$ (constant)

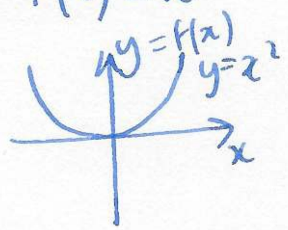
Proof suppose there is a,b with $f(a) \neq f(b)$. Then $\exists\, c \in (a,b)$ with $f'(c) = \dfrac{f(b)-f(a)}{b-a} \neq 0$ # □

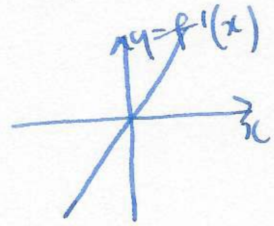
Monotonicity suppose f is differentiable on (a,b):

if $f'(x) > 0$ for all $x \in (a,b)$ then f is increasing on (a,b)

$f'(x) < 0$ decreasing

Example ① $f(x) = x^2$ $\qquad f'(x) = 2x$





increasing on $(0,\infty)$

decreasing on $(-\infty,0)$

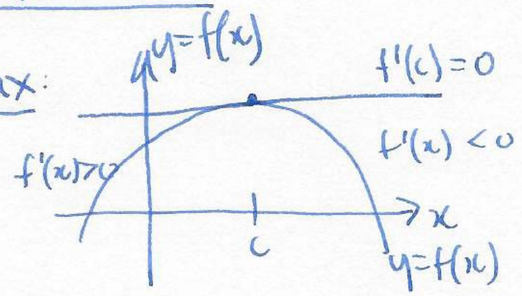
② $f(x) = x^2 - 2x - 3$

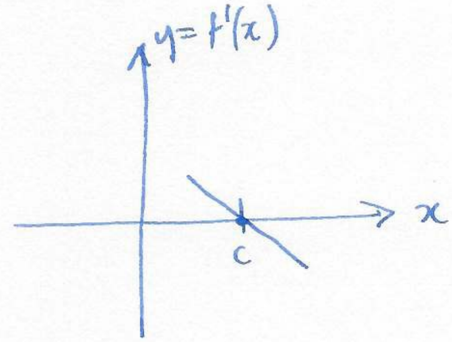
$f'(x) = 2x-2 \qquad f'(x) > 0$ when $2x-2>0$

$x > 1$.

First derivative test

local max:





$f'(x)$ goes from positive to negative $\Rightarrow$ local max.