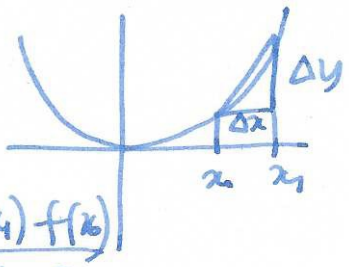
§ 3.4 Rates of change

recall: average rate of change $= \frac{\Delta y}{\Delta x} = \frac{f(x_1)-f(x_0)}{x_1-x_0}$

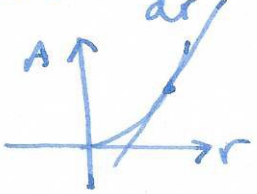


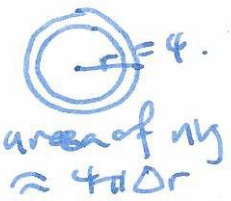
(instantaneous) rate of change $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_1 \to x_0} \frac{f(x_1)-f(x_0)}{x_1-x_0}$

observation: if Δx is small, we can use average rate of change to approximate actual rate of change, and vice-versa.

Example: area of circle is πr^2

calculate rate of change of area wrt radius: $\frac{dA}{dr} = 2\pi r$





e.g. $\left.\frac{dA}{dr}\right|_{r=2} = 4\pi$ $\left.\frac{dA}{dr}\right|_{r=5} = 10\pi$

for small h, $f'(x_0) \approx \frac{f(x_0+h)-f(x_0)}{h}$

or $f(x_0+h) \approx f(x_0) + hf'(x_0)$

Example stopping distance in feet given by $F(s) = 1.1s + 0.05s^2$ (s speed in mph) calculate stopping distance when $s=30$.

$F(30) = 1.1(30) + 0.05(30)^2 = 78$

calculate rate of change of stopping distance wrt speed when $s=30$

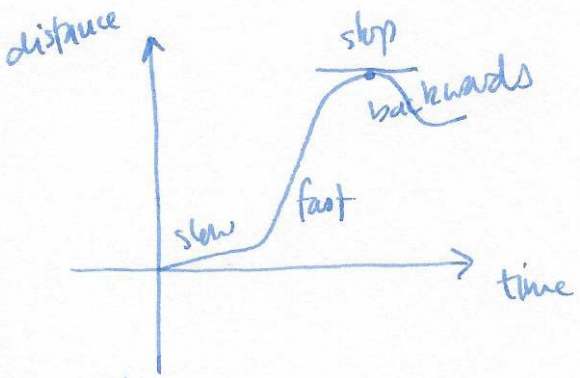
$F'(s) = 1.1 + 0.1s$ $\quad F'(30) = 1.1 + 0.1(30) = 4.1$ ft/mph.

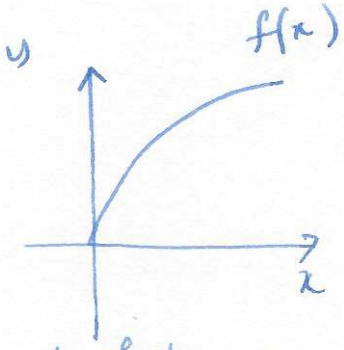
estimate stopping distance at $s=31$ (using above info):

$F(s+h) \approx F(s) + hF'(s)$

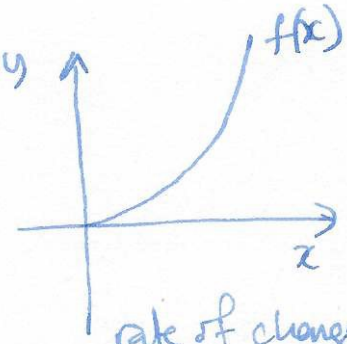
$F(31) \approx F(30) + 1 \cdot F'(30) = 78 + 1 \times 4.1 = 82.1$

On the interpretation of graphs

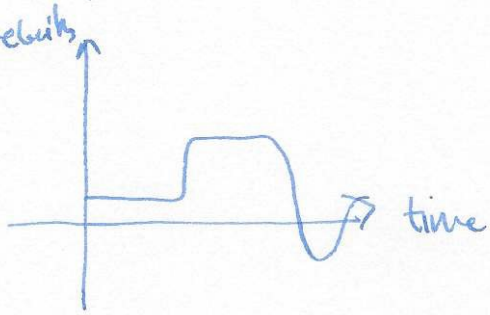


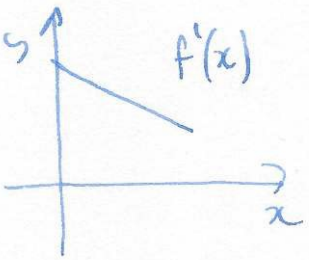


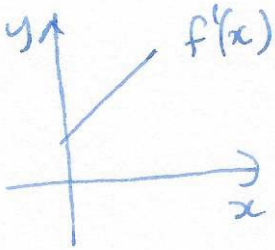
rate of change decreasing



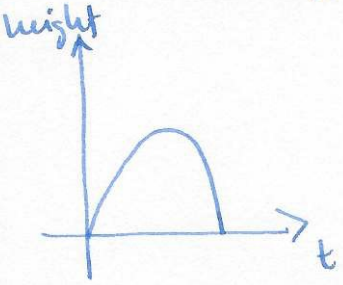
rate of change increasing







Motion under gravity



$s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$

$s'(t) = v(t) = v_0 - gt$

$s''(t) = v'(t) = a(t) = -g$ (constant) $\quad g = 9.8 m/s^2$ or $32 ft/s^2$.

$s_0 = s(0)$ = height at $t=0$

$v_0 = v(0)$ = speed at $t=0$

Q: when is the maximum height?

A: when $v(t)=0$. $\quad v_0 - gt = 0 \Rightarrow t = \frac{v_0}{g}$ so $s\left(\frac{v_0}{g}\right) = s_0 + \frac{v_0^2}{g} - \frac{1}{2} g \frac{v_0^2}{g^2}$

$= s_0 + \frac{1}{2} \frac{v_0^2}{g}$.

Example throw a stone upwards at 10m/s from a height of 2m.

What is max height?

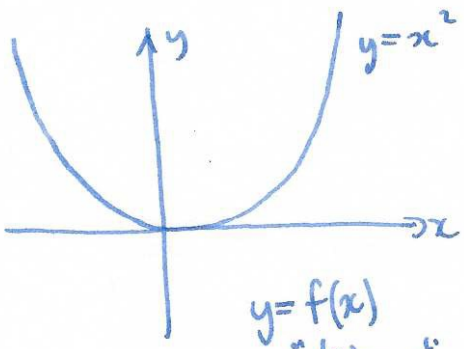
$s(t) = 2 + 10t - \frac{1}{2} g t^2$

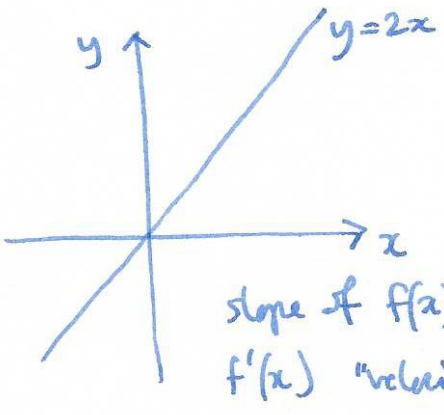
$v(t) = 10 - gt$

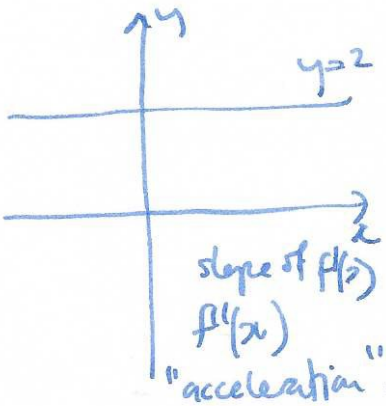
$v(t) = 0 \Rightarrow t = \frac{10}{g} \approx 1 \quad s(1) = 2 + 10 - 5 = 7m$.

Q: if I can throw a stone 10m high, how fast can I throw it?

§3.5. Higher derivatives







Example $f(x) = xe^x$

$f'(x) = xe^x + e^x$

$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$

$f'''(x) = f^{(3)}(x) = xe^x + e^x + 2e^x = xe^x + 3e^x$ etc

Example acceleration due to gravity