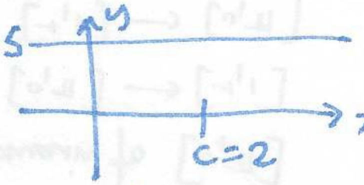
<u>Def</u>

Def$^{\underline{n}}$ Let f be a function defined on an interval containing c, but not necessarily defined at c. We say "the limit of $f(x)$ as x approaches c is equal to L" if $|f(x)-L|$ becomes arbitrarily small as x gets close to c.

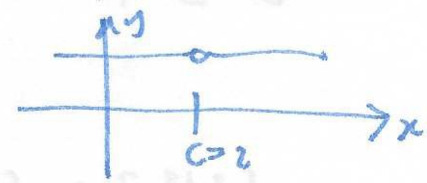
notation: $\lim\limits_{x\to c} f(x) = L$ or $f(x) \to L$ as $x \to c$

we also "$f(x)$ converges to L as x tends to c"

<u>Examples</u> a) $f(x) = 5$, $c = 2$



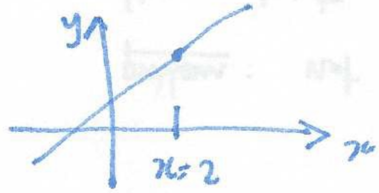
b) $f(x) = \frac{5x}{x}$, $c = 2$



want to show: $|f(x)-5|$ close to 0 if x close to 2

$|f(x)-5| = |5-5| = 0$ for all $x \neq 2$, so this is true.

c) $\lim\limits_{x\to 2} 2x+1 = 5$



want to show $|f(x)-5|$ close to zero when x close to 2

$|f(x)-5| = |2x+1-5| = |2x-4| = 2|x-2|$

x close to 2 $\Leftrightarrow$ $|x-2|$ small.

<u>useful facts</u>

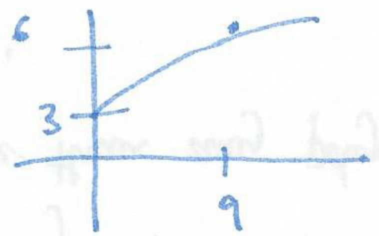
<u>Thm</u> for any constants k, c $\quad \lim\limits_{x\to c} k = k$

$\lim\limits_{x\to c} x = c$

Investigating limits: try
- drawing a picture
- calculate close values
- algebra

<u>Example</u> $\displaystyle\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$ problem: can't plug in $x=9$, get $\frac{0}{0}$

. draw picture



looks like $f(x) = 6$

. calculate:

x	$\frac{x-9}{\sqrt{x}-3}$
8.9	5.98329
9.1	6.016
8.99	5.99833
9.01	6.00166

. algebra: difference of two squares

$x-9 = (\sqrt{x}-3)(\sqrt{x}+3)$

$$\frac{x-9}{\sqrt{x}-3} = \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3}.$$

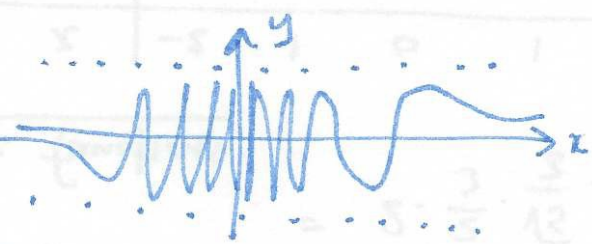
$$= \sqrt{x}+3 \quad (x \neq 9)$$

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3} = \lim_{x\to 9}\sqrt{x}+3 = 6$$

<u>Bad example</u>: no limit

$f(x) = \sin\left(\frac{1}{x}\right)$

no limit near $x = 0$

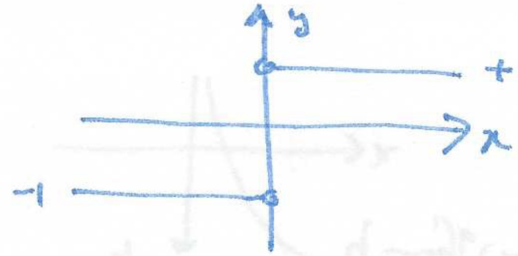


<u>note</u>: $f\left(\frac{1}{2\pi n}\right) = \sin(2\pi n) = 0$

$f\left(\frac{1}{2\pi n + \frac{\pi}{2}}\right) = \sin\left(2\pi n + \frac{\pi}{2}\right) = 1.$

One sided limits

Example $f(x) = \frac{x}{|x|}$



$$f(x) = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$$

sometimes useful to distinguish left from right limits

notation: $\lim_{x \to 0+} f(x)$ means right limit (only consider $x > 0$)

$\lim_{x \to 0-} f(x)$ means left limit (only consider $x < 0$)

note: in order for the two sided limit $\lim_{x \to c} f(x)$ to exist, the right limit must equal the left limit: $\lim_{x \to c+} f(x) = \lim_{x \to c-} f(x)$

Example $f(x) = \frac{x}{|x|}$

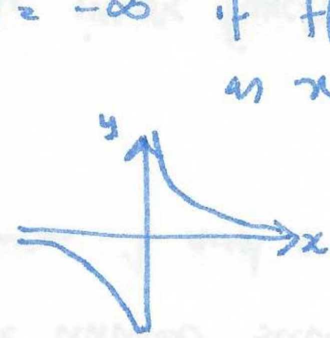
$$\left.\begin{array}{l} \lim_{x \to 0+} f(x) = 1 \\ \lim_{x \to 0-} f(x) = -1 \end{array}\right\} 1 \neq -1 \text{ so } \lim_{x \to 0} f(x) \text{ DNE}$$

Infinite limits

We say $\lim_{x \to c} f(x) = +\infty$ if $f(x)$ becomes arbitrarily large and positive as $x \to c$

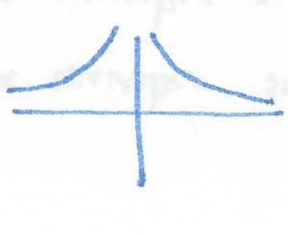
$\lim_{x \to c} f(x) = -\infty$ if $f(x)$ " " " negative as $x \to c$

Example $f(x) = \frac{1}{x}$



$$\left.\begin{array}{l} \lim_{x \to 0+} \frac{1}{x} = +\infty \\ \lim_{x \to 0-} \frac{1}{x} = -\infty \end{array}\right\} \text{different! so } \lim_{x \to 0} \frac{1}{x} \text{ DNE}$$

Example $f(x) = \frac{1}{x^2}$



$$\lim_{x \to 0+} \frac{1}{x^2} = +\infty = \lim_{x \to 0-} \frac{1}{x^2} = +\infty.$$

so $\lim_{x \to 0} \frac{1}{x^2} = +\infty$