# Math 231 Calculus I

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office 1S-222 office hours M 12:20 - 2:15
W 1:25 - 2:15

- math tutoring: 1S - 214
- students with disabilities

Text: Calculus, Rogawski 2nd ed.

HW: webworks.

## §1.2 Linear and quadratic functions

recall: function

input set ⟼ output set
domain        range

examples: $f: \mathbb{R} \to \mathbb{R}$ ($\mathbb{R}$ = real numbers)
$x \mapsto x^2$ or $f(x) = x^2$ ( description of function )

e.g. $0 \mapsto 0$
$1 \mapsto 1$
$-1 \mapsto 1$
$2 \mapsto 4$ etc.

notation $f: \mathbb{R} \to \mathbb{R}$

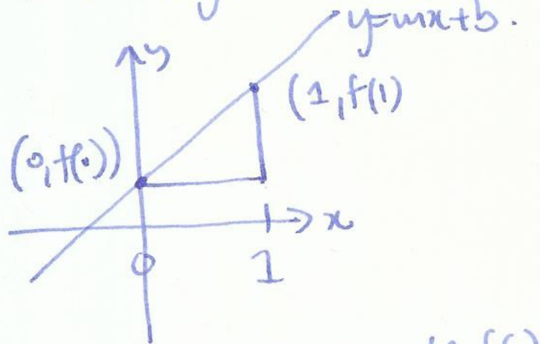
name, domain, range

examples $+: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$
$(a,b) \mapsto a+b$

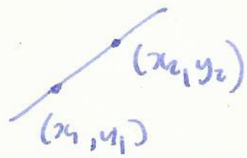
evaluation at 0 : $\{\text{functions } f: \mathbb{R} \to \mathbb{R}\} \mapsto \mathbb{R}$
$f \mapsto f(0)$

key property: every input goes to a unique output

A linear function $f : \mathbb{R} \to \mathbb{R}$ has the form $f(x) = mx + b$ ($m, b$ "constants", i.e. don't depend on $x$). The graph of a linear function is a straight line.

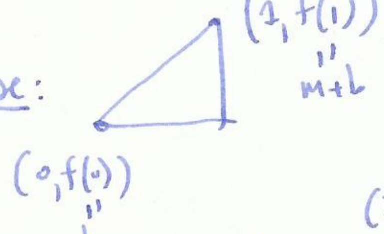



$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$



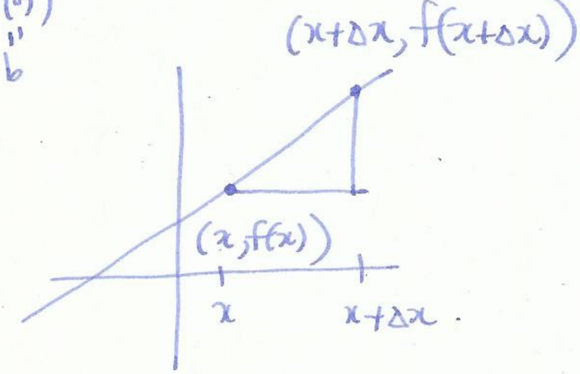

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

<u>special case</u>:




$$\frac{\Delta y}{\Delta x} = \frac{f(1) - f(0)}{1 - 0} = \frac{m + b - b}{1} = m.$$

<u>general case</u>:



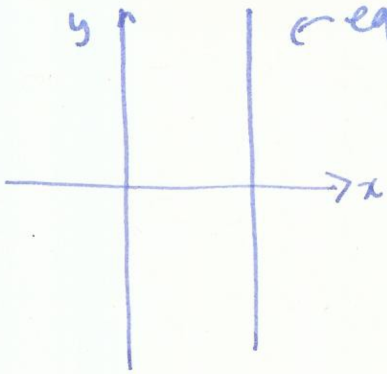

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{x + \Delta x - x} = \frac{m(x + \Delta x) + b - (mx + b)}{\Delta x}$$

$$= \frac{\cancel{mx} + \Delta x . m + \cancel{b} - \cancel{mx} - \cancel{b}}{\Delta x} = m.$$

<u>useful fact</u>: a straight line has constant slope everywhere.

<u>observations</u>:

- $|m|$ large steep slope
- $m = 0$ horizontal line
- $m > 0$ increasing (from left to right)
- $m < 0$ decreasing ( " )
- vertical lines not graphs of functions.




← equation of vertical line is $x = c$

equation vs function

equation ← relation between variables, e.g. $x = c$, $x^2+y^2 = 1$

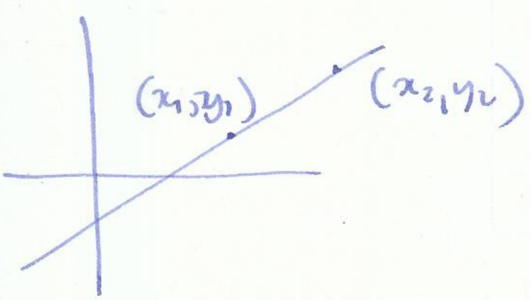
function ← a map from one set to another e.g. $f: \mathbb{R} \to \mathbb{R}$.

the graph of $f: \mathbb{R} \to \mathbb{R}$ is $y = f(x)$ (an equation!)
but not every equation comes from a function.

to deal with any straight line use the general linear equation
$ax + by = c$ (at least one of $a, b$ not zero).

e.g. $x = c$ set $b = 0, a = 1$.

useful technique : find equation of line through two points.

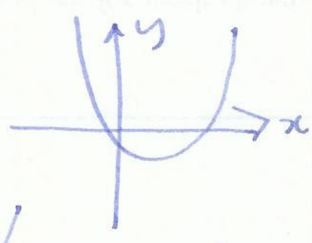



- find slope : $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- line $y - y_1 = m(x - x_1)$.

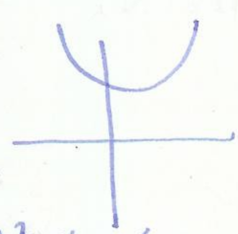
Quadratic functions

given by $f(x) = ax^2 + bx + c$ ($a, b, c$ constants, do not depend on $x$)

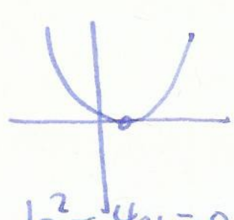
graphs are parabolas



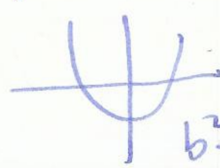

at most two distinct real solutions to $f(x) = 0$
given by $f(x) = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.



$b^2 - 4ac < 0$



$b^2 - 4ac = 0$



$b^2 - 4ac > 0$

## useful techniques

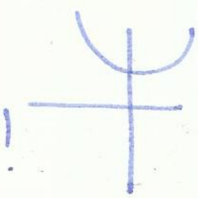
- factorization: $ax^2+bx+c = a(x-r_1)(x-r_2)$ $r_1, r_2$ solutions or roots.
- complete the square: any quadratic function can be written as

$(x+a)^2+b$ example

$x^2+2x+3$

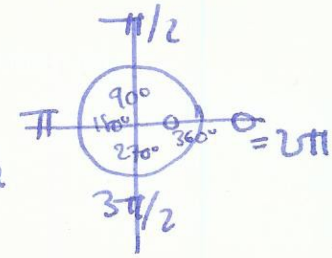
$(x+1)^2 \quad +2$

$x^2+2x+1+2$ no sol'ns!



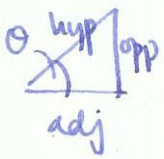
## §1.4 Trig functions

- angles vs radians radians win.

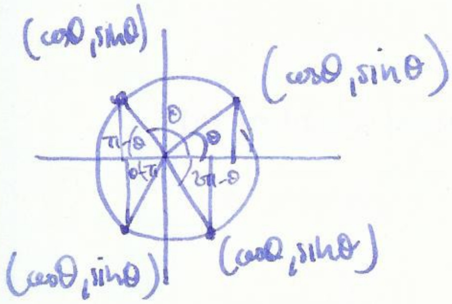



angle in radians = distance travelled around the unit circle

- right angled triangles




$\sin\theta = \dfrac{opp}{hyp}$, $\cos\theta = \dfrac{adj}{hyp}$ } can extend these functions to be defined for all $\theta \in \mathbb{R}$.



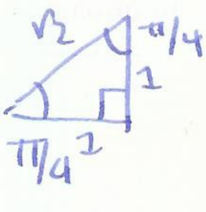

useful facts: $\sin(-\theta) = -\sin(\theta)$ (odd function)

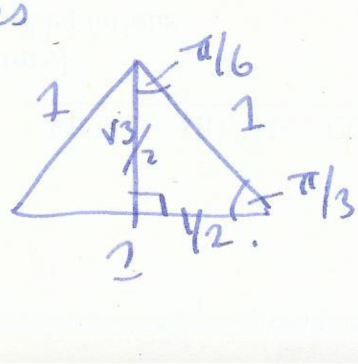
$\cos(-\theta) = \cos(\theta)$ (even function)

special values

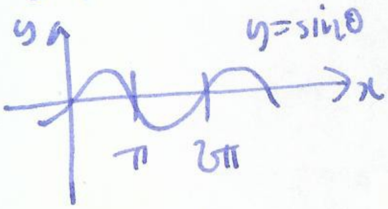
| $\theta$ | $0$ | $\pi/2$ | $\pi/4$ | $\pi/3$ | $\pi/6$ |
|---|---|---|---|---|---|
| $\sin\theta$ | $0$ | $1$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | $1/2$ |
| $\cos\theta$ | $1$ | $0$ | $\sqrt{2}/2$ | $1/2$ | $\sqrt{3}/2$ |

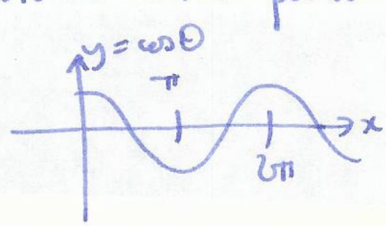
from special triangles



- graphs of $\sin\theta, \cos\theta$ are periodic with period $2\pi$.

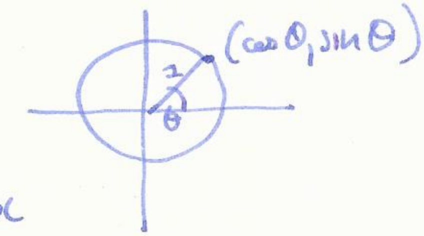



<u>other trig functions</u> $\tan(x) = \dfrac{\sin(x)}{\cos(x)} = \dfrac{opp}{adj}$

$\operatorname{cosec}(x) = \dfrac{1}{\sin x}$ $\sec(x) = \dfrac{1}{\cos(x)}$ $\cot(x) = \dfrac{1}{\tan(x)} = \dfrac{\cos(x)}{\sin(x)}$.

<u>Trig identities</u>




<u>Pythagorean identity</u>: $\cos^2 x + \sin^2 x = 1$

$$\frac{\cos^2 x + \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \longleftrightarrow \cot^2 x + 1 = \operatorname{cosec}^2 x$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \longleftrightarrow 1 + \tan^2 x = \sec^2 x.$$

<u>Double angle</u>: $\sin 2\theta = 2\sin\theta\cos\theta$

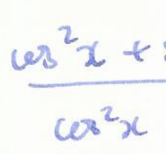
$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

<u>Addition</u>: $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

special case: (shift). $\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$.

<u>Example</u> suppose $\sin\theta = \frac{2}{3}$ find $\cos\theta$, $\tan\theta$, $\sin 2\theta$.




$\cos\theta = \dfrac{\sqrt{5}}{3}$ $\tan\theta = \dfrac{2}{\sqrt{5}}$ $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \cdot \dfrac{2}{3} \cdot \dfrac{\sqrt{5}}{3} = \dfrac{4\sqrt{5}}{9}$.