Math 231 Calculus 1 Fall 14 Midterm 3b

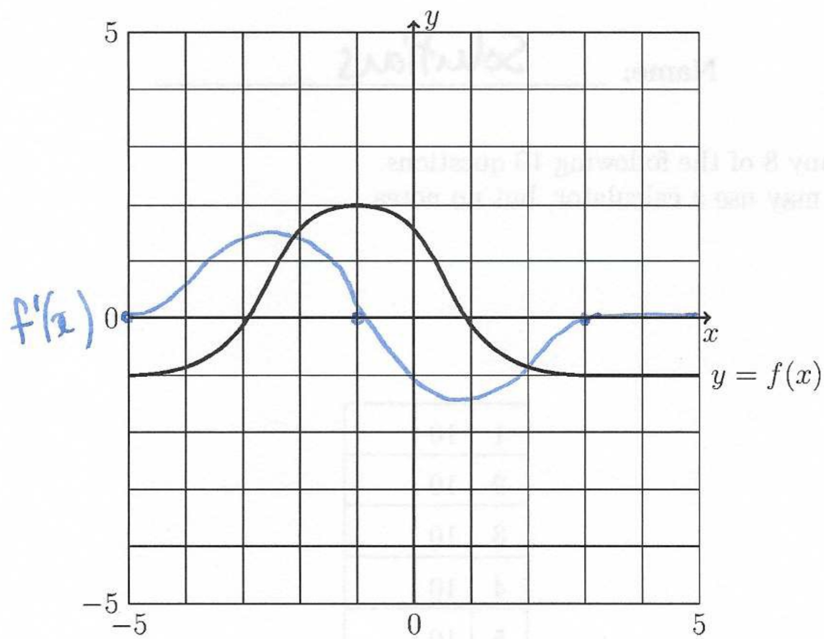
Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) (10 points) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f(x) > 0$. $(-3, 1)$

(b) Label all regions where $f'(x) < 0$. $(-1, 5)$ (or $(-1, 3)$ acceptable)

(c) What is $\lim_{x\to\infty} f(x)$? -1

(d) What is $\lim_{x\to\infty} f'(x)$? 0

(e) Sketch a graph of $f'(x)$ on the figure.

(2) (10 points) Consider the function $f(x) = \dfrac{1}{x^2 - x - 2}$. $= \dfrac{1}{(x-2)(x+1)}$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) vertical asymptotes: $x = 2$, $x = -1$
horizontal asymptotes: $y = 0$

b) $f'(x) = -(x^2 - x - 2)^{-2} \cdot (2x - 1)$ solve $f'(x) = 0$ $x = \frac{1}{2}$

c)

$2x-1$	$-$		$-$		$+$		$+$
$-(x^2-x-2)^2$	$-$		$-$		$-$		$-$
		-1		$1/2$		2	
$f'(x)$	$+$		$+$		$-$		$-$

increasing: $(-\infty, -1) \cup (-1, \frac{1}{2})$
decreasing: $(1/2, 2) \cup (2, \infty)$.

(3) (10 points) Consider the function $f(x) = x\ln(x) - 4x$.

(a) Find all critical points of the function.

(b) Use the second derivative test to attempt to classify them.

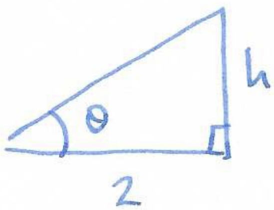
a) $f'(x) = \ln(x) + x\cdot\frac{1}{x} - 4$

solve $f'(x) = 0$ $\quad \ln(x) = 3 \quad x = e^3$

b) $f''(x) = \frac{1}{x}$

$f''(e^3) = \frac{1}{e^3} > 0 \Rightarrow$ local min

(4) (10 points) A hot air balloon is released from the ground a distance of 2km away. When you see the balloon at an angle of $\pi/6$ radians, it is rising at a rate of 0.1 radians/hour. How fast is the balloon rising?



$$\frac{h}{2} = \tan\theta \qquad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

$$\frac{1}{2}\frac{dh}{dt} = \sec^2\theta \, \frac{d\theta}{dt}$$

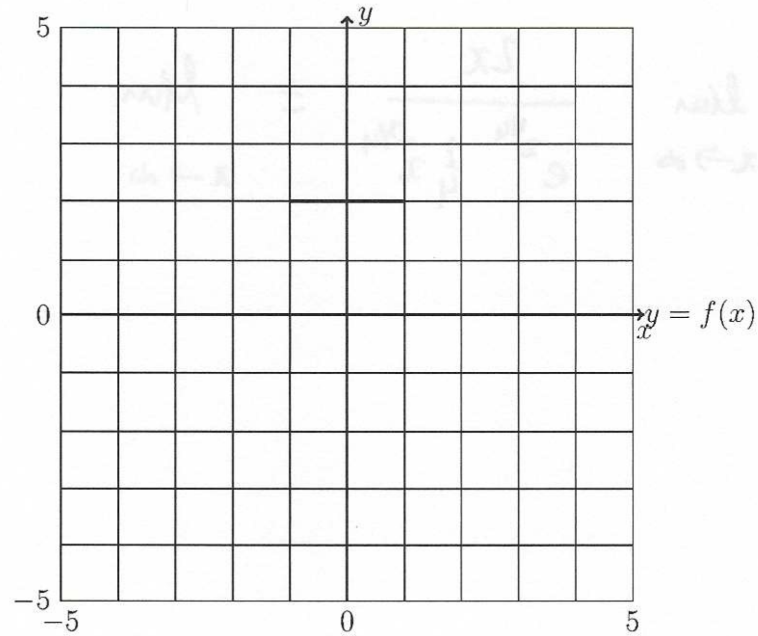
$$\frac{dh}{dt} = 2 \cdot \frac{4}{3} \cdot 0.1 = \frac{8}{30} \text{ km/hour}.$$

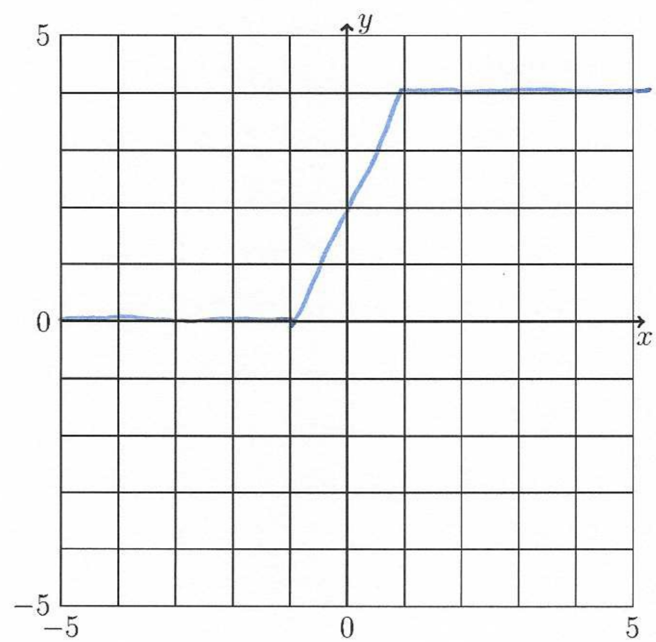
(5) (10 points) Find
$$\lim_{x\to 0} \frac{\ln(x+1)-x}{1-\cos 2x}.$$

l'H: $\displaystyle\lim_{x\to 0} \frac{\frac{1}{x+1}-1}{2\sin 2x}$

l'H: $\displaystyle\lim_{x\to 0} \frac{-(x+1)^{-2}\cdot 1}{4\cos(4x)} = \frac{-1}{4}$

(6) (10 points) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.





(7) (10 points) Which function grows faster, x^2 or $e^{\sqrt[4]{x}}$? Justify your answer. (Hint: take a limit.)

$$\lim_{x \to \infty} \frac{x^2}{e^{x^{1/4}}} \overset{L'H}{=} \lim_{x \to \infty} \frac{2x}{e^{x^{1/4}} \, \frac{1}{4} x^{-3/4}} = \lim_{x \to \infty} \quad 8x$$

(8) (10 points) Find the indefinite integral

$$\int 3\cos(x) - 2e^x \, dx.$$

$+ 3\sin(x) - 2e^{x} + C$

(9) (10 points) Evaluate the definite integral

$$\int_1^9 \frac{x-2}{\sqrt{x}}\,dx.$$

$$\int_1^9 x^{1/2} - 2x^{-1/2}\,dx = \left[\frac{2}{3}x^{3/2} - 4x^{1/2}\right]_1^9$$

$$= \frac{2}{3}27 - \underbrace{4.3}_{12} - \left(\frac{2}{3} - 4\right).$$

$$= 18 - 12 - \frac{2}{3} + 4 = 9\frac{1}{3}$$

(10) Find the area under the graph $y = 2x^3 + 1$ between $x = 0$ and $x = 1$.

$$\int_0^1 2x^3+1 \, dx = \left[\frac{1}{2}x^4 + x\right]_0^1$$

$$= \frac{1}{2} + 1 = 1\tfrac{1}{2} = \frac{3}{2}.$$