Math 231 Calculus 1 Fall 14 Midterm 3a

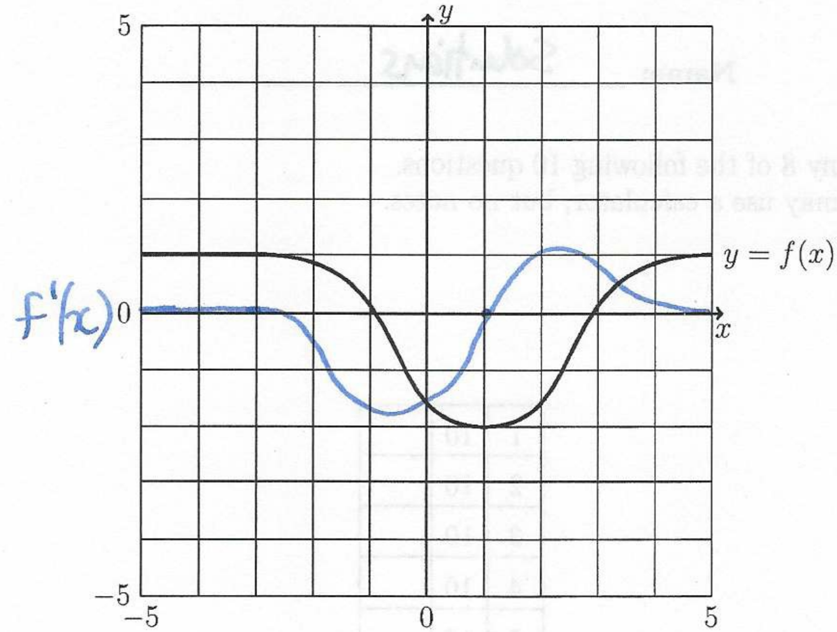
Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) (10 points) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f(x) < 0$. $(-1, 3)$

(b) Label all regions where $f'(x) > 0$. $(1, 5)$ 4

5

(c) What is $\lim_{x\to-\infty} f(x)$? 1

(d) What is $\lim_{x\to-\infty} f'(x)$? 0

5 (e) Sketch a graph of $f'(x)$ on the figure.

(2) (10 points) Consider the function $f(x) = \dfrac{1}{x^2 + x - 2}$. $= \dfrac{1}{(x+2)(x-1)}$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) vertical asymptotes : $x = -2, 1$
horizontal asymptotes : $y = 0$

b) $f'(x) = -(x^2+x-2)^{-2} \cdot (2x+1)$ solve $f'(x) = 0$ $2x+1 = 0$ $x = -1/2$

c)

		-2		$-1/2$		1	
$2x+1$	$-$		$-$		$+$		$+$
$-(x^2+x-2)^2$	$-$		$-$		$-$		$-$
$f'(x)$	$+$		$+$		$-$		$-$

increasing : $(-\infty, -2) \cup (-2, -\frac{1}{2})$

decreasing : $(-\frac{1}{2}, 1) \cup (1, \infty)$

(3) (10 points) Consider the function $f(x) = x\ln(x) - 3x$.

(a) Find all critical points of the function.

(b) Use the second derivative test to attempt to classify them.

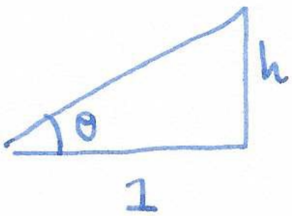
a) $f'(x) = \ln(x) + x \cdot \frac{1}{x} - 3$

solve $f'(x) = 0$: $\ln(x) = 2$ $\quad x = e^2$

b) $f''(x) = \frac{1}{x}$

$f''(e^2) = \frac{1}{e^2} > 0$ local min.

(4) (10 points) A hot air balloon is released from the ground a distance of 1km away. When you see the balloon at an angle of $\pi/4$ radians, it is rising at a rate of 0.2 radians/hour. How fast is the balloon rising?



$$\frac{h}{1} = \tan\theta$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sec^2\frac{\pi}{4} = \frac{4}{2} = 2.$$

$$\frac{dh}{dt} = \sec^2\theta \, \frac{d\theta}{dt}$$

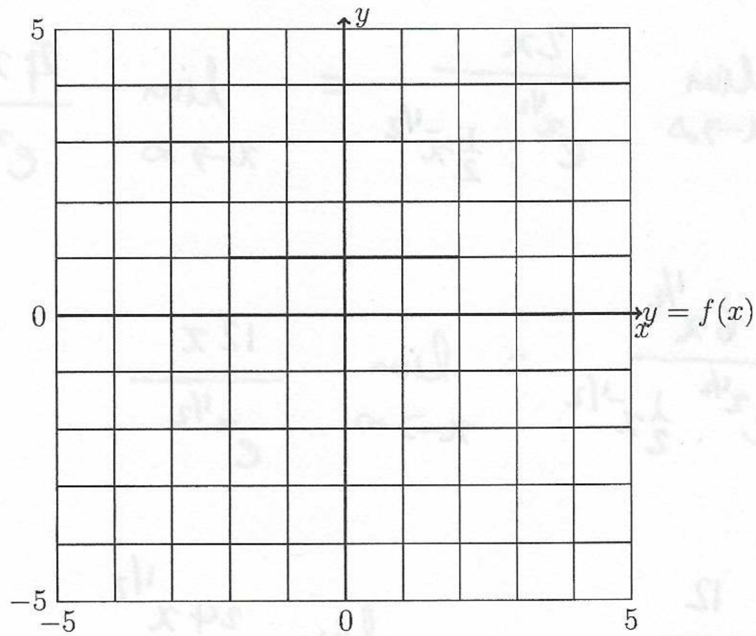
$$= \sec^2\left(\frac{\pi}{4}\right) 0.2 = 0.4 \text{ km}/\text{hour}.$$

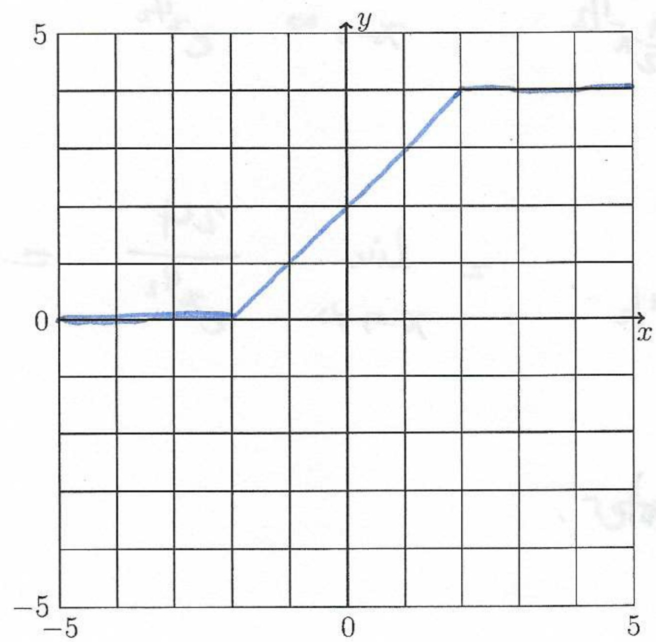
(5) (10 points) Find
$$\lim_{x \to 0} \frac{\ln(x+1) - x}{\cos(3x) - 1}.$$

l'H: $\lim\limits_{x \to 0} \dfrac{\frac{1}{x+1} - 1}{-3\cancel{\cos}\sin(3x)}$

l'H: $\lim\limits_{x \to 0} \dfrac{-(x+1)^{-2} \cdot 1}{-9\cos(3x)} = \dfrac{-1}{-9} = \dfrac{1}{9}$

(6) (10 points) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.





(7) (10 points) Which function grows faster, x^2 or $e^{\sqrt[2]{x}}$? Justify your answer. (Hint: take a limit.)

$$\lim_{x\to\infty} \frac{x^2}{e^{x^{1/2}}} \underset{l'H}{=} \lim_{x\to\infty} \frac{2x}{e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{4x^{3/2}}{e^{x^{1/2}}}$$

$$l'H: = \lim_{x\to\infty} \frac{6x^{1/2}}{e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{12x}{e^{x^{1/2}}}$$

$$l'H: = \lim_{x\to\infty} \frac{12}{e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{24x^{1/2}}{e^{x^{1/2}}}$$

$$l'H = \lim_{x\to\infty} \frac{12x^{-1/2}}{e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{24}{e^{x^{1/2}}} = 0$$

so $e^{x^{1/2}}$ grows faster.

(8) (10 points) Find the indefinite integral

$$\int 4e^x + 2\sin(x)\ dx.$$

$$4e^x - 2\cos(x) + c$$

(9) (10 points) Evaluate the definite integral

$$\int_1^9 \frac{x+2}{\sqrt{x}}\, dx.$$

$$\int_1^9 x^{1/2} + 2x^{-1/2}\, dx = \left[\frac{2x^{3/2}}{3} + 4x^{1/2} \right]_1^9$$

$$= \frac{2}{3} \cdot 27 + \frac{4 \cdot 3}{12} - \left(\frac{2}{3} + 4 \right)$$

$$= 18 + 12 - \frac{2}{3} - 4 = 26 - \frac{2}{3} = 25\tfrac{1}{3}.$$

(10) Find the area under the graph $y = 2x^3 + 2$ between $x = 0$ and $x = 1$.

$$\int_0^1 2x^3 + 2\,dx = \left[\frac{1}{2}x^4 + 2x \right]_0^1$$

$$= \frac{1}{2} + 2 = 2\tfrac{1}{2}.$$