Sample midterm 3 Solutions

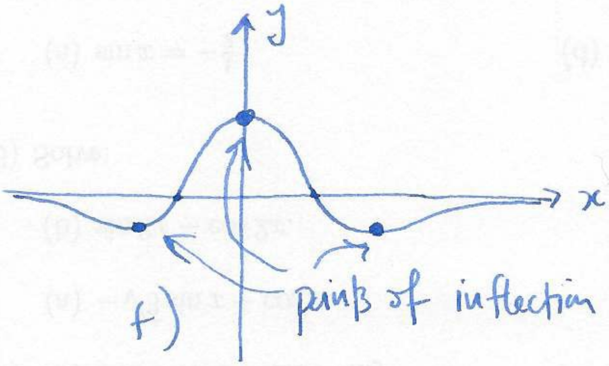
Q1 a) $[-5,-1) \cup (1,5]$

b) $(-1,1)$

c) 0

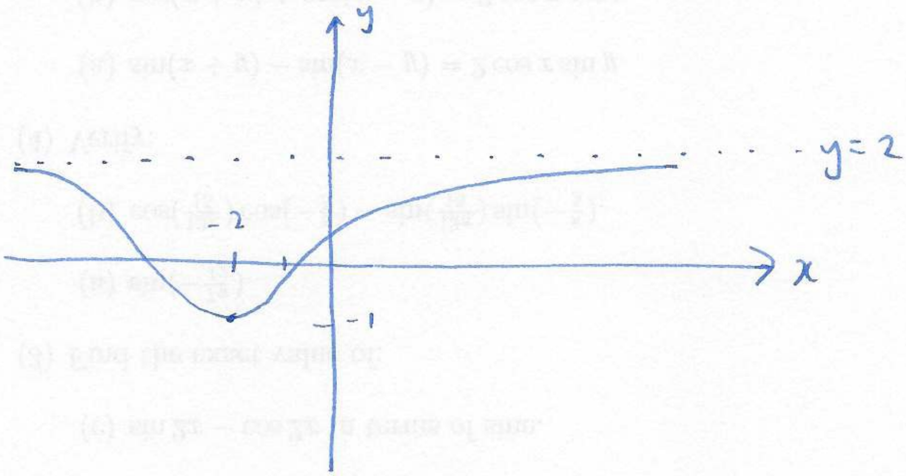
d) 0

e)



f) points of inflection ($f''(x)$ changes sign)

Q2



Q3 a) vertical asymptotes: $16-x^2=0 \Rightarrow x = \pm 4$.

horizontal asymptotes: $\dfrac{x^2}{-x^2} = -1 \qquad y=-1$.

b) $$f'(x) = \frac{(16-x^2)2x-(-2x)x^2}{(16-x^2)^2} = \frac{2x(16-x^2+x^2)}{(16-x^2)^2} = \frac{32x}{(16-x^2)^2}.$$

c) increasing for $x>0$
decreasing for $x<0$

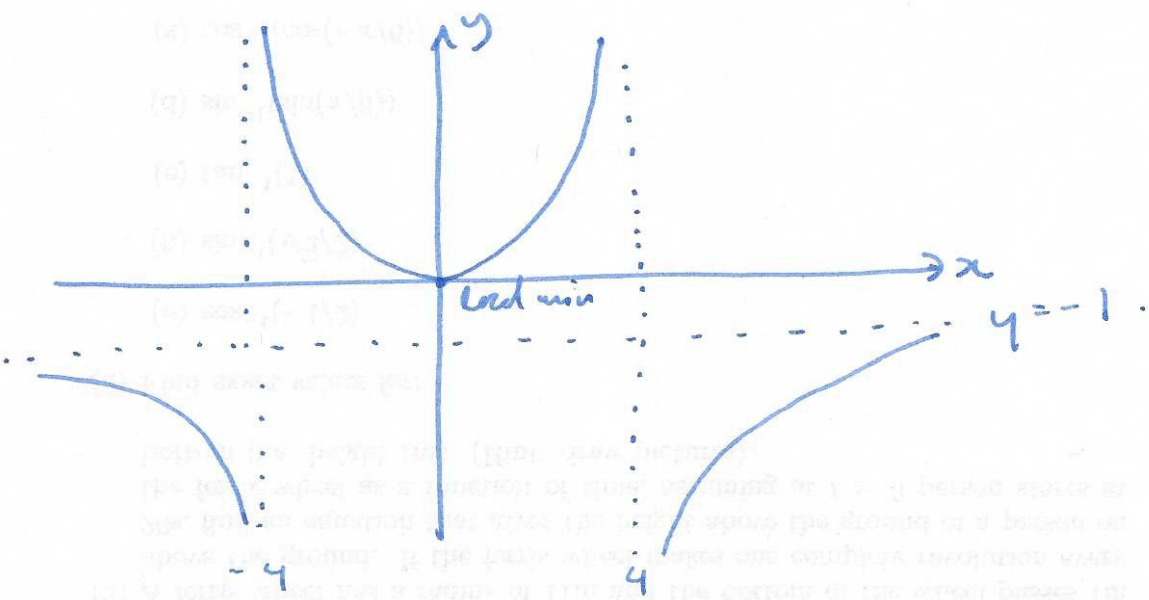
critical points $x=0$
and asymptotes $x=\pm 4$.

d) $f''(x) = \dfrac{(16-x^2)^2 \cdot 32 - 2(16-x^2)\cdot(-2x)\cdot 32x}{(16-x^2)^4}$

$= \dfrac{32(16-x^2)\left[16-x^2+4x^2\right]}{(16-x^2)^4} = \dfrac{32(16+3x^2)}{(16-x^2)^3}$.

$f''(0) > 0 \Rightarrow$ local min.

e)



Q4 $g(x) = (x^2+2x)e^x$

a) $g'(x) = (2x+2)e^x + (x^2+2x)e^x = (x^2+4x+2)e^x$

solve $g'(x)=0$ $\quad e^x \neq 0 \quad x^2+4x+2=0 \quad x = \dfrac{-4 \pm \sqrt{16-8}}{2} = -2 \pm \sqrt{2}$

sign of $g'(x)$: $+$ at $-2-\sqrt{2}$ $-$ at $-2+\sqrt{2}$ $+$

local max $\quad$ local min

b) increasing on $(-\infty, -2-\sqrt{2}) \cup (-2+\sqrt{2}, \infty)$.

decreasing on $(-2-\sqrt{2}, -2+\sqrt{2})$.

(3)

c) $g''(x) = (2x+4)e^x + (x^2+4x+2)e^x = (x^2+6x+6)e^x$

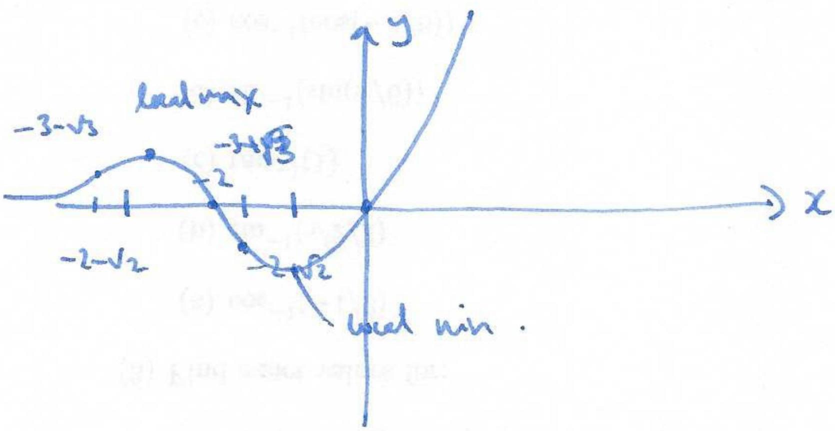
$x = \dfrac{-6 \pm \sqrt{36-24}}{2} = -3 \pm \sqrt{3}$

sign of $g''(x)$: + (−3−√3) − (−3+√3) +

d) concave up: $(-\infty, -3-\sqrt{3}) \cup (-3+\sqrt{3}, \infty)$

concave down: $(-3-\sqrt{3}, -3+\sqrt{3})$.

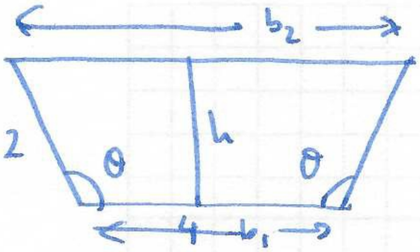
e)



$g(x) = 0 \Rightarrow x = 0, -2$

Q5 $f'(x) = \dfrac{1}{e^{2x}+1} > 0$ increasing, so max at 3.

Q6



area of trapezoid $= \frac{1}{2}(b_1+b_2)h$

$= \frac{1}{2}\left(4+4+4\sin\left(\underbrace{\theta-\frac{\pi}{2}}_{\phi}\right)\right)2\cos\left(\underbrace{\theta-\frac{\pi}{2}}_{\phi}\right)$

set $\theta - \frac{\pi}{2} = \phi$

$A(\phi) = (4+2\sin\phi)\,2\cos\phi$

$A'(\phi) = 2\cos\phi \cdot 2\cos\phi + (4+2\sin\phi)(-2\sin\phi)$

$= 4\cos^2\phi - 8\sin\phi - 4\sin^2\phi$.

$$= 4(1-\sin^2\phi) - 8\sin\phi - 4\sin^2\phi.$$

$$= 4 - 8\sin\phi - 8\sin^2\phi$$

solve $A'(\phi)=0$: $\quad -4(2\sin^2\phi + 2\sin\phi - 1) = 0$

$$\sin\phi = \frac{-2 \pm \sqrt{4+8}}{4} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}.$$

$$\phi = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) + \frac{\pi}{2}.$$

Q7 a) $\displaystyle \lim_{x\to\infty} \frac{+6x+2}{\sqrt{2x^3-4}} \sim \lim_{x\to\infty} \frac{6x}{\sqrt{2}\, x^{3/2}} = \lim_{x\to\infty} \frac{6}{\sqrt{2}}\frac{1}{\sqrt{x}} = 0.$

b) $\displaystyle \lim_{x\to 0+} \sqrt[3]{x}\, \ln(x) = \lim_{x\to 0+} \frac{\ln(x)}{x^{-1/3}}$

l'H: $\displaystyle = \lim_{x\to 0+} \frac{1/x}{-\frac{1}{3}x^{-4/3}} = \lim_{x\to 0+} -3x^{-1+4/3} = \lim_{x\to 0+} -3x^{1/3} = 0.$

c) $\displaystyle \lim_{x\to 0} \frac{1}{2x^2} - \frac{1}{1-\cos(2x)} = \lim_{x\to 0} \frac{1-\cos(2x)-2x^2}{2x^2(1-\cos(2x))}$

l'H: $\displaystyle = \lim_{x\to 0} \frac{\sin(2x)\cdot 2 - 4x}{4x(1-\cos(2x)) + 2x^2\cdot\sin(2x)\cdot 2}$

l'H: $\displaystyle = \lim_{x\to 0} \frac{\cos(2x)\cdot 4 - 4}{4(1-\cos(2x)) + 4x(\sin(2x)\cdot 2) + 4x\sin(2x)\cdot 2 + 2x^2\cos(2x)\cdot 4}.$

$$= \lim_{x\to 0} \frac{4\cos(2x) - 4}{\sin(2x)\cdot(8x+8x) + \cos(2x)\cdot(-4+8x^2) + 4}.$$

$$l'H: \lim_{x\to 0} \frac{-8\sin(2x)}{\underbrace{2\cos(2x)\cdot 16x + \sin(2x)\cdot 16 - \sin(2x)\cdot 2(-4+8x^2) + \cos(2x)\cdot 16x}_{\to\ 48x\cos(2x) + \sin(2x)\cdot(\underbrace{16+8}_{24} - 16x^2)}}.$$

$$l'H: \lim_{x\to 0} \frac{-16\cos(2x)}{48\cos(2x) - 48x\sin(2x)\cdot 2 + 2\cos(2x)\cdot(24-16x^2) + \sin(2x)(-32x)}.$$

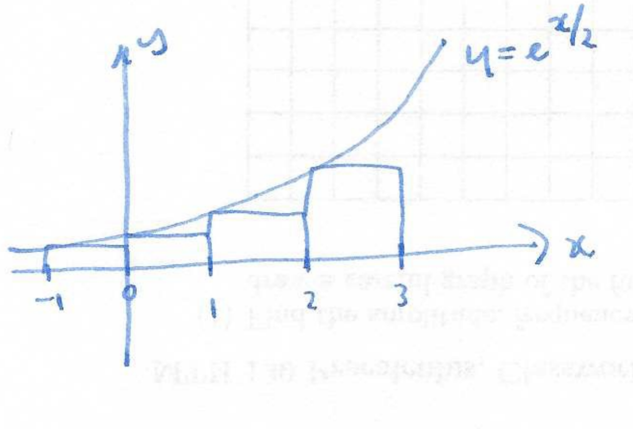
$$= \frac{-16}{48+2\cdot 24} = -\frac{1}{6}.$$

d) $$\lim_{x\to 0} \frac{\cos x - \cos 3x}{2x\sin x}$$

$$l'H: \lim_{x\to 0} \frac{-\sin x + 3\sin 3x}{2\sin x + 2x\cos x}$$

$$l'H: \lim_{x\to 0} \frac{-\cos x + 9\cos 3x}{2\cos x + 2\cos x - 2x\sin x} = \frac{-1+9}{2+2} = 2.$$

Q8



approx area:

$$e^{-1/2} + e^{0} + e^{1/2} + e^{1} \approx 5.974$$

under estimate.

Q9 a) $\int \frac{x^2+3x-1}{x}\,dx = \int x+3-\frac{1}{x}\,dx = \frac{1}{2}x^2+3x-\ln|x|+c$

b) $\int_{-2}^{2}|x|\,dx = \int_{-2}^{0}-x\,dx + \int_{0}^{2}x\,dx$

$= \left[-\frac{1}{2}x^2\right]_{-2}^{0} + \left[\frac{1}{2}x^2\right]_{0}^{2} = \frac{1}{2}\cdot 4+\frac{1}{2}\cdot 4 = 4.$

c) $\int_{1}^{2} 2x^{-1/4}\,dx = \left[2.\ \frac{4}{3}x^{+3/4}\right]_{1}^{2} = \frac{8}{3}\left(2^{3/4}-1\right).$

d) $\int_{0}^{x}\frac{1}{t+1}\,dx = \Big[\ln|1+t|\Big]_{0}^{x} = \ln|1+x|.$

e) $\int \frac{1}{1+2x^2}\,dx$ ~~tn~~ ~~set u=~~ use $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

so $\frac{d}{dx}\left(\tan^{-1}(\sqrt{2}x)\right) = \frac{1}{1+2x^2}\cdot\sqrt{2}$

so $= \frac{1}{\sqrt{2}}\tan^{-1}(x)+c.$

Q10 $\frac{dx}{dt} = (t+1)^{-3}$ $\quad x(t) = -\frac{1}{2}(t+1)^{-2}+c$

$t=0\ x=0:\ -\frac{1}{2}\cdot 1+c=0 \quad c=\frac{1}{2}.$

$x(t) = \frac{1}{2}-\frac{1}{2}(t+1)^{-2} \to \frac{1}{2}$ as $t\to\infty$ so does not get to $x=1$.