## Math 231 Calculus 1 Fall 14 Midterm 2~~a~~ b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| | 80 | |

| | |
|---|---|
| Midterm 2 | |
| Overall | |

(1) (10 points) Find the derivative of the function $f(x) = xe^{-3x^2}$.

$$e^{-3x^2} + x \cdot e^{-3x^2} \cdot -6x$$

$$= e^{-3x^2} - 6x^2 e^{-3x^2}$$

(2) (10 points) Find the derivative of $f(x) = \dfrac{\sin(x) - 1}{\cos(3x) + 1}$.

$$\frac{(\cos(3x)+1)(\sin(x)-1)' - (\sin(x)-1)(\cos(3x)+1)'}{(\cos(3x)+1)^2}$$

$$= \frac{(\cos(3x)+1)\cos(x) - (\sin(x)-1)(-\sin(3x)\cdot 3)}{(\cos(3x)+1)^2}$$

(3) (10 points) Find the derivative of the function $f(x) = \tan^{-1}\left(\frac{2}{x}\right)$.

$$\frac{1}{1+\left(\frac{2}{x}\right)^2} \cdot \left(\frac{2}{x}\right)' \qquad \frac{2}{x} = 2x^{-1}$$

$$= \frac{1}{1+\frac{4}{x^2}} \cdot -2x^{-2}$$

$$= \frac{-2}{x^2+4}$$

(4) (10 points) Find the derivative of the function $f(x) = \ln(1 - 3x^4)$.

$$\frac{1}{1-3x^4} \cdot \left(-12x^3\right)$$

$$= \frac{-12x^3}{1-3x^4}$$

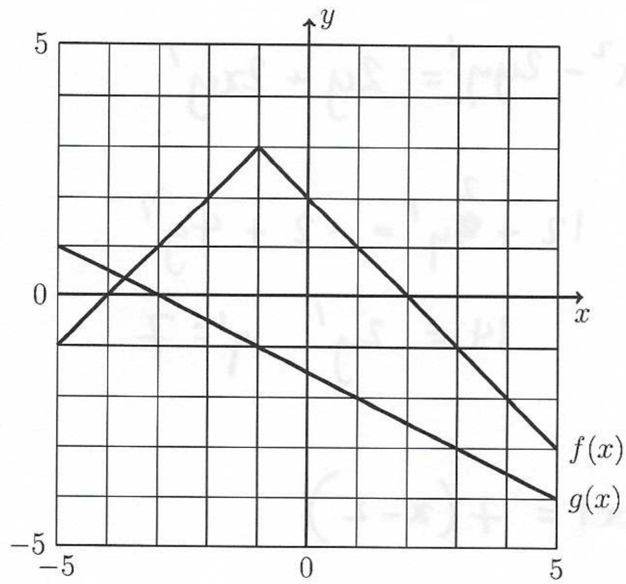
(5) (10 points) Find the second derivative of the function $f(x) = \sqrt{3x+2}$.

$f(x) = (3x+2)^{1/2}$

$f'(x) = \frac{1}{2}(3x+2)^{-1/2} \cdot 3 = \frac{3}{2}(3x+2)^{-1/2}$

$f''(x) = -\frac{3}{4}(3x+2)^{-3/2} \cdot 3 = -\frac{9}{4}(3x+2)^{-3/2}$

(6) (10 points) The graph of the functions $f$ and $g$ are shown below.




(a) Let $h(x) = f(x)g(x)$. Find $h'(0)$.

$h'(x) = f'(x)g(x) + f(x)g'(x)$ $\qquad$ $h'(0) = f'(0)g(0) + f(0)g'(0)$

$-1 \cdot -\frac{3}{2} + 2 \cdot -\frac{1}{2} \quad = \frac{1}{2}$

(b) Let $h(x) = f(x)/g(x)$. Find $h'(1)$.

$h'(x) = \dfrac{g'(x)f(x) - f'(x)g(x)}{(g(x))^2}$ $\qquad$ $h'(1) = \dfrac{g'(1)f(1) - f'(1)g(1)}{(g(1))^2} = \dfrac{-1/2 \cdot 1 - (-1)\cdot(-2)}{(-2)^2}$

$= \dfrac{-5/2}{4} = -\frac{5}{8}$

(c) Let $h(x) = f(g(x))$. Find $h'(-2)$.

$h'(x) = f'(g(x)) \cdot g'(x)$ $\qquad$ $h'(-2) = f'(g(-2)) \cdot g'(-2)$

$= f'(-1/2) \cdot \frac{-1}{2} = (-1) \cdot \frac{-1}{2} = +\frac{1}{2}$

(7) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $x^3 - y^2 = 2xy + 9$ at the point $(2, -1)$.

$$3x^2 - 2yy' = 2y + 2xy'$$

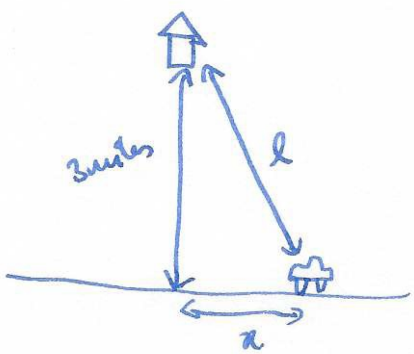
at $(2,-1)$:

$$12 + \overset{2}{\cancel{8}}y' = -2 + 4y'$$

$$14 = 2y' \quad y' = 7$$

$$y + 1 = 7(x - 2)$$

(8) (10 points) A house lies 3 miles from the freeway, on a road perpendicular to the freeway. If you drive on the freeway at 60mph, how fast is your distance to the house changing when you are three miles past the junction?




$$l^2 = 3^2 + x^2 = 9 + x^2$$

$$2l \frac{dl}{dt} = 2x \frac{dx}{dt} \qquad \frac{dx}{dt} = 60$$

$$x = 3$$

$$l = \sqrt{9+9} = 3\sqrt{2}$$

$$\frac{dl}{dt} = \frac{3}{3\sqrt{2}} 60 = \frac{60}{\sqrt{2}} = 30\sqrt{2}$$

(9) (10 points) Use linear approximation to estimate $\sqrt{62}$. What is the percentage error in your approximation?

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f(x+\Delta x) \approx f(x) + \Delta x f'(x)$

$8^2 = 64$

$f(62) \approx f(64) + (-2)f'(64)$

$= 8 + -2 \cdot \frac{1}{2} \frac{1}{\sqrt{64}} = 8 - \frac{1}{8} = 7\frac{7}{8}$

percentage error $= \dfrac{\left| 7\frac{7}{8} - \sqrt{62} \right|}{\sqrt{62}} \approx 0.013\ \%$

(10) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is decreasing. On the lower set of axes sketch $f'(x)$.