Math 231 Calculus 1 Fall 14 Midterm 2a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of the function $f(x) = xe^{-4x^2}$.

$$e^{-4x^2} + x \cdot e^{-4x^2} \cdot -8x$$

$$= e^{-4x^2} - 8x^2 e^{-4x^2}$$

(2) (10 points) Find the derivative of $f(x) = \dfrac{\sin(x) - 1}{\cos(2x) + 1}$.

$$\frac{(\cos(2x)+1)(\sin(x)-1)' - (\sin(x)-1)(\cos(2x)+1)'}{(\cos(2x)+1)^2}$$

$$= \frac{(\cos(2x)+1)\cos(x) - (\sin(x)-1)(-\sin(2x).2)}{(\cos(2x)+1)^2}$$

(3) (10 points) Find the derivative of the function $f(x) = \tan^{-1}\left(\frac{3}{x}\right)$.

$$\frac{1}{1+\left(\frac{3}{x}\right)^2} \cdot \left(\frac{3}{x}\right)' \qquad \frac{3}{x} = 3x^{-1}$$

$$= \frac{1}{1+\frac{9}{x^2}} \cdot -3x^{-2}$$

$$= \frac{-3}{x^2+9}$$

(4) (10 points) Find the derivative of the function $f(x) = \ln(1 - 2x^4)$.

$$\frac{1}{1-2x^4} \cdot (-8x^3)$$

$$= \frac{-8x^3}{1-2x^4}$$

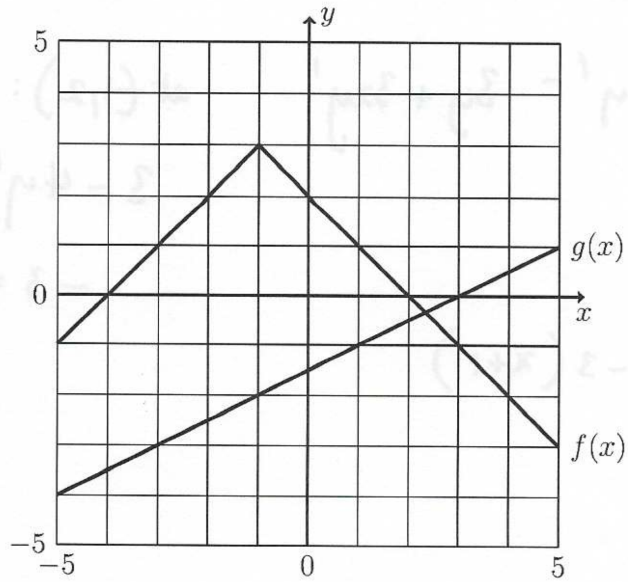
(5) (10 points) Find the second derivative of the function $f(x) = \sqrt{2x+3}$.

$f(x) = (2x+3)^{1/2}$

$f'(x) = \frac{1}{2}(2x+3)^{-1/2} \cdot 2 = (2x+3)^{-1/2} = \frac{1}{\sqrt{2x+3}}$

$f''(x) = -\frac{1}{2}(2x+3)^{-3/2} \cdot 2 = -(2x+3)^{-3/2}$

(6) (10 points) The graph of the functions f and g are shown below.



(a) Let $h(x) = f(x)g(x)$. Find $h'(2)$.

$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(2) = f'(2)\,g(2) + f(2)\,g'(2) \quad = 1/2$

$-1 \quad \cancel{0}\,-\tfrac{1}{2} \quad 0 \cdot 1/2$

(b) Let $h(x) = f(x)/g(x)$. Find $h'(0)$.

$h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$ $\qquad h'(0) = \dfrac{g(0)f'(0) - f(0)g'(0)}{g(0)^2}$

$h'(0) = \dfrac{-\frac{3}{2}\cdot -1 - 2 \cdot 1/2}{(-3/2)^2} = \dfrac{+1/2}{9/4} = +\dfrac{2}{9}$

(c) Let $h(x) = f(g(x))$. Find $h'(-2)$.

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(-2) = f'(g(-2)) \cdot g'(-2) = f'\left(-\frac{5}{2}\right) \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2}$

(7) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $x^3 - y^2 = 3xy + 1$ at the point $(-1, 2)$.

$3x^2 - 2yy' = 3y + 3xy'$

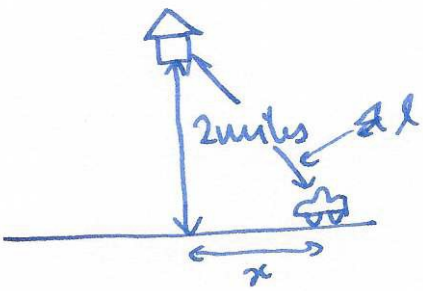
at $(-1,2)$:

$3 - 4y' = 6 - 3y'$

$-3 = y'$

$y - 2 = -3(x+1)$

(8) (10 points) A house lies 2 miles from the freeway, on a road perpendicular to the freeway. If you drive on the freeway at 60mph, how fast is your distance to the house changing when you are two miles past the junction?



$$\ell^2 = 2^2 + x^2 = 4 + x^2$$

$$2\ell \frac{d\ell}{dt} = 2x \frac{dx}{dt}$$

$$\frac{dx}{dt} = 60$$

$$x = 2$$

$$\ell = \sqrt{4+4} = 2\sqrt{2}$$

$$\frac{d\ell}{dt} = \frac{2}{2\sqrt{2}} 60 = \frac{60}{\sqrt{2}} = 30\sqrt{2}$$

(9) (10 points) Use linear approximation to estimate $\sqrt{79}$. What is the percentage error in your approximation?

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2} x^{-1/2}$$

$$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$$

$$9^2 = 81$$

$$f(79) \approx f(81) + f'(81)(-2)$$

$$9 + \frac{1}{2}\frac{1}{\sqrt{81}} \cdot -2 = 9 - \frac{1}{9} = \cancel{8\tfrac{2}{3}} \cdot 8\frac{8}{9}$$

$$\text{percentage error} = \frac{\left| \overset{8/9}{8\cancel{\tfrac{2}{3}}} - \sqrt{79} \right|}{\sqrt{79}} \times 100 \approx \cancel{[illegible]}$$

0.0078%.

(10) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is increasing. On the lower set of axes sketch $f'(x)$.