MTH 231 Sample midterm 2 solutions

Q1 a) $2xe^{-5x^2} + x^2 e^{-5x^2} \cdot -10x$

$= 2xe^{-5x^2} - 10x^3 e^{-5x^2}$

b) $\dfrac{(2-\sec(3x)) \cdot \frac{1}{2}x^{-1/2} - (\sec(3x)\tan(3x).3)\sqrt{x}}{(2-\sec(3x))^2}$

c) $x^{4x} = e^{4x\ln(x)}$ $\quad (x^{4x})' = e^{4x\ln(x)} \cdot \left(4x \cdot \frac{1}{x} + 4\ln(x)\right)$

$= x^{4x}(4 + 4\ln(x))$.

d) $\dfrac{1}{\tan(4x)} \cdot \dfrac{1}{\sec^2(4x)} \cdot 4$

e) $\dfrac{1}{1+\left(\frac{2}{\sqrt{x}}\right)^2} \cdot 2 - \frac{1}{2}x^{-3/2} = \dfrac{-x^{-3/2}}{1 + 4/x}$

f) $\dfrac{-1}{\sqrt{1-(1-x)^2}} \cdot -1 = \dfrac{1}{\sqrt{1-(1-x)^2}} = \dfrac{1}{\sqrt{2x-x^2}}$.

Q2 a) $2 \cdot e^{-5x^2} + 2x \cdot e^{-5x^2} \cdot -10x - 30x^2 e^{-5x^2} - 10x^3 e^{-5x^2} \cdot -10x$.

$= e^{-5x^2}(2 - 50x^2 + 100x^4)$

b) $= \left(\dfrac{1 - \frac{1}{2}\sec(3x) - 3x\sec(3x)\tan(3x)}{\sqrt{x}\,(2-\sec(3x))^2}\right)'$

$= \dfrac{\sqrt{x}'(2-\sec(3x))^2\left[-\frac{1}{2}\sec(3x)\tan(3x).3 - 3\sec(3x)\tan(3x) - 3x(\sec(3x)\tan(3x))'\right]}{x(2-\sec(3x))^4} - \ldots$

$$= \sqrt{x}\left(2-\sec(3x)\right)^2\left[\frac{9}{2}\sec(3x)\tan(3x) - 3x\left(\sec(3x)\tan(3x)\tan(3x).3 + \sec(3x)\sec^2(3x).3\right)\right]$$
$$- \left[\frac{1}{2}x^{-1/2}\left(2-\sec(3x)\right)^2 + \sqrt{x}\,2\left(2-\sec(3x)\right).-\sec(3x)\tan(3x).3\right]\left(1-\frac{1}{2}\sec(3x) - 3x\sec(3x)\tan(3x)\right)$$

$$\sqrt{x}\left(2-\sec(3x)\right)^2.$$

$$= x\left(2-\sec(3x)\right)^2\sec(3x)\tan(3x)\left[-\frac{9}{2}\tan(3x) - 9x\tan^2(3x) + 3\sec^2(3x)\right]$$
$$- \left[\frac{1}{2}\left(2-\sec(3x)\right)^2 + x\,6\left(2-\sec(3x)\right)\left(-\sec(3x)\tan(3x)\right)\right]\left(1-\frac{1}{2}\sec(3x) - 3x\sec(3x)\tan(3x)\right)$$

$$x\left(2-\sec(3x)\right)^2.$$

c) $\left(e^{4x\ln(x)}\cdot\left(4+4\ln(x)\right)\right)' =$

$$= e^{4x\ln(x)}\left(4+4\ln(x)\right)^2 + e^{4x\ln(x)}\cdot\frac{4}{x}.$$

d) $\left(\frac{4}{\tan(4x)\sec^2(4x)}\right)' = \left(\frac{4\cos^3(x)}{\sin(x)}\right)' = 4\,\frac{\sin x\cdot 3\cos^2 x\cdot(-\sin x) + \cos(x)\cos^3 x}{\sin^2 x}.$

$$= 4\,\frac{\cos^4(x)\left(-3\sin^2 x + \cos^2 x\right)}{\sin^2 x}.$$

d) $\left(4\cot(4x)\cos^2(4x)\right)' = 4\left(-\operatorname{cosec}(4x).4\right)\cos^2(4x)$
$$+ 4\cot(4x).2\cos(4x).(-\sin(4x).4.$$

e) $$\frac{\left(1+\frac{4}{x}\right)\left(\frac{3}{2}x^{-5/2}\right) - \left(-4x^{-2}\right)\left(-x^{-3/2}\right)}{\left(1+\frac{4}{x}\right)^2}.$$

f) $\left((2x-x^2)^{-1/2}\right)' = -\frac{1}{2}(2x-x^2)^{-3/2}(2-2x)$.

Q3 a) $h'(x) = f'(x)g(x) + f(x)g'(x)$.

$h'(-3) = f'(3)g(3) + f(3)g'(3)$

$= 1 \cdot -\frac{1}{2} + 0 \cdot -\frac{1}{2} = -\frac{1}{2}$

b) $h'(x) = f'(g(x)) \cdot g'(x)$

$h'(4) = f'(g(4)) \cdot g'(4) = f'(-1) \cdot -\frac{1}{2} = -\frac{1}{4}$.

Q4 $4x^2 + 2y^2 = 18 \qquad 8x + 4yy' = 0$

at $(-2,1)$: $-16 + 4y' = 0 \quad y' = 4$

$y - 1 = 4(x+2)$

Q5 $y^2 + x2yy' + 4xy^2 + 2x^2 2yy' = -\cos(xy)\cdot(y + xy')$.

$y'(2xy + 4x^2y + \cos(xy)\cdot x) = -y^2 - 4xy^2 - \cos(xy)y$

$y' = -\dfrac{y^2 + 4xy^2 + \cos(xy)y}{2xy + 4x^2y + x\cos(xy)}$

Q6 $V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2 \quad r = \sqrt{\frac{A}{4\pi}}$ $\Bigg\}$ $V = \frac{4}{3}\pi \frac{A^{3/2}}{4^{3/2}\pi^{3/2}} = \frac{A^{3/2}}{6\sqrt{\pi}}$

$\frac{dV}{dt} = \frac{1}{6\sqrt{\pi}} \frac{3}{2} A^{1/2} \frac{dA}{dt}$ $\qquad \frac{dV}{dt} = 10$

$r = 10 \Rightarrow A = 400\pi$

$$10 = \frac{1}{4\sqrt{\pi}}\sqrt{400\pi}\,\frac{dA}{dt} \qquad \frac{dA}{dt} = \frac{10}{20}\cdot 4 = 2\,cm/sec.$$

Q7 $f(x) = x^{1/3}$ $\qquad f(27) = 3$

$f'(x) = \frac{1}{3}x^{-2/3}$ $\qquad f'(27) = \frac{1}{27}$

$$f(26) \approx f(27) + (-1)f'(27) = 3 - \frac{1}{27} \simeq 2\frac{26}{27}.$$