Math 231 Calculus 1 Fall 14 Midterm 1b

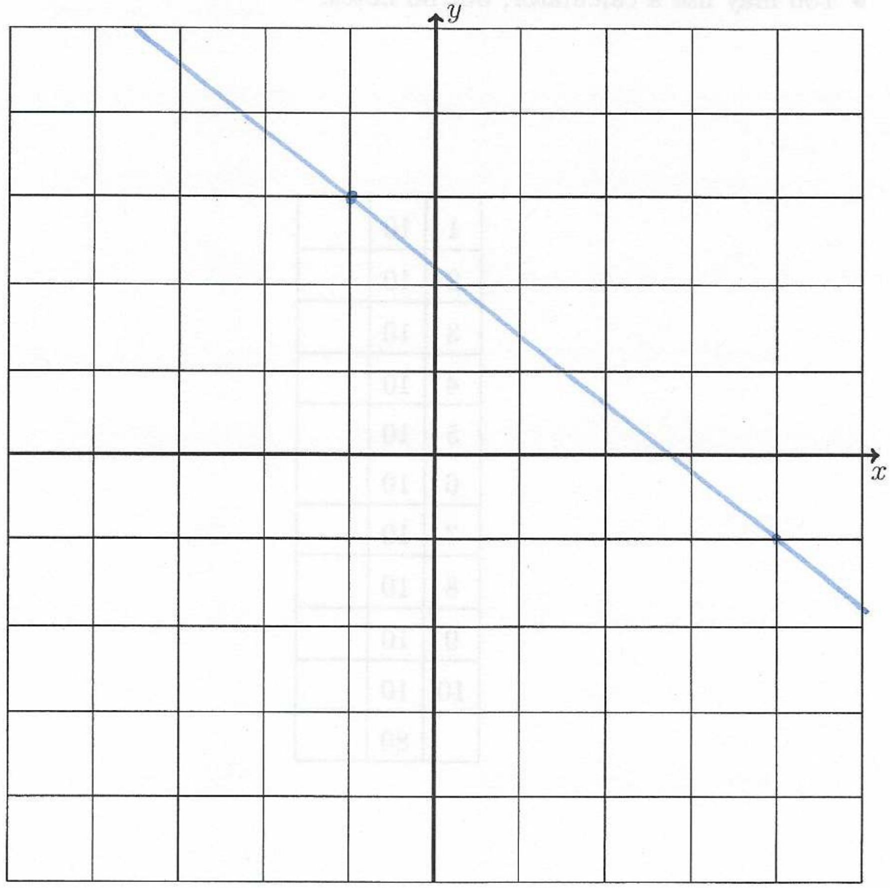
Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-1, 3)$ and $(4, -1)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



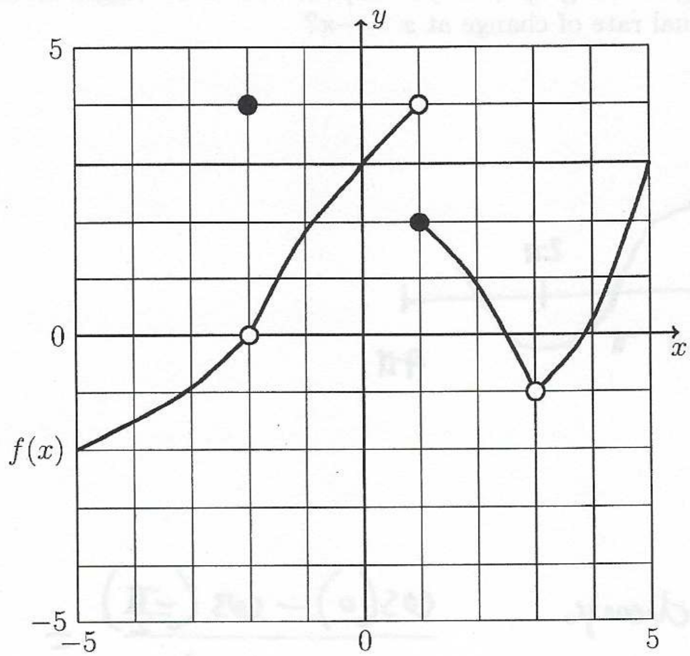
slope $\frac{-1-3}{4-(-1)} = \frac{-4}{5}$

$$\boxed{y - 3 = -\frac{4}{5}(x+1)}$$

$$y = -\frac{4}{5}x + 3 - 4/5$$

$$y = -\frac{4}{5}x + \frac{11}{5}$$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x\to 3} f(x)$ -1

(b) $\lim_{x\to -2-} f(x)$ 0

(c) $\lim_{x\to -2+} f(x)$ 0

(d) $\lim_{x\to -2} f(x)$ 0

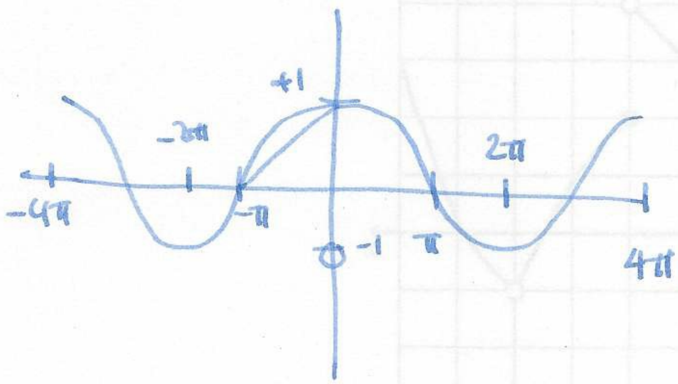
(e) $\lim_{x\to 1+} f(x)$ 2

(f) $\lim_{x\to 1} f(x)$ DNE

(3) (10 points) Sketch the graph of $f(x) = \cos(x/2)$.

(a) What is the average rate of change from $x = -\pi$ to $x = 0$?

(b) Looking at the graph, do you expect this to be bigger or smaller than the actual rate of change at $x = -\pi$?



a) average rate of change $\frac{\cos(0) - \cos(\frac{-\pi}{2})}{0-(-\pi)} = \frac{1-0}{\pi} = \frac{1}{\pi}$

b) average rate of change smaller than actual rate of change at $x = -\pi$.

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 0} \frac{\sin 2x}{-5x}$$

$2x = \theta$

$$\lim_{\theta \to 0} \frac{\sin\theta}{-5(\theta/2)} = -\frac{2}{5} \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = -\frac{2}{5}$$

(5) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$$

$$\lim_{x \to 2} \frac{(x-2)(x+3)}{x-2} = \lim_{x \to 2} x+3 = 5$$

(6) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 3} \frac{x-3}{x-\sqrt{x+6}}$$

$$\lim_{x\to 3} \frac{(x-3)}{(x-\sqrt{x+6})} \frac{(x+\sqrt{x+6})}{(x+\sqrt{x+6})} = \lim_{x\to 3} \frac{(x-3)(x+\sqrt{x+6})}{x^2-x-6}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+\sqrt{x+6})}{(x-3)(x+2)} = \lim_{x\to 3} \frac{x+\sqrt{x+6}}{x+2} = \frac{3+\sqrt{9}}{5} = \frac{6}{5}.$$

(7) (10 points) Use the limit definition of the derivative to differentiate $f(x) = 3x^2 - x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + h^2 - x - h - \cancel{3x^2} + \cancel{x}}{h} = \lim_{h \to 0} 6x + h - 1 = 6x - 1$$

(8) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{3-x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{3-(x+h)} - \frac{1}{3-x}}{h}$$

$$= \lim_{h\to 0} \frac{(\cancel{3}-x)-(\cancel{3}-x-h)}{h(3-x-h)(3-x)} = \lim_{h\to 0} \frac{h}{h(3-x-h)(3-x)} = \lim_{h\to 0} \frac{1}{(3-x-h)(3-x)}$$

$$= \frac{1}{(3-x)^2}$$

(9) (10 points) Find the horizontal asymptotes of $f(x) = \frac{\sqrt{x^2+2}}{5x-1}$.

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{5x-1} = \lim_{x\to\infty} \frac{\sqrt{1+2/x^2}}{5-1/x} = \frac{1}{5}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{5x-1} = \lim_{x\to\infty} \frac{\sqrt{1+2/x^2}}{1/x-5} = -\frac{1}{5}$$

horizontal asymptotes: $y = \frac{1}{5}$

$y = -\frac{1}{5}$.

(10) (10 points) Sketch the graph of a function for which $f(1) = 2$, f is increasing for $x < 0$ and decreasing for $x > 0$, and $\lim_{x \to \infty} f(x) = -2$.