Math 231 Calculus 1 Fall 14 Midterm 1a

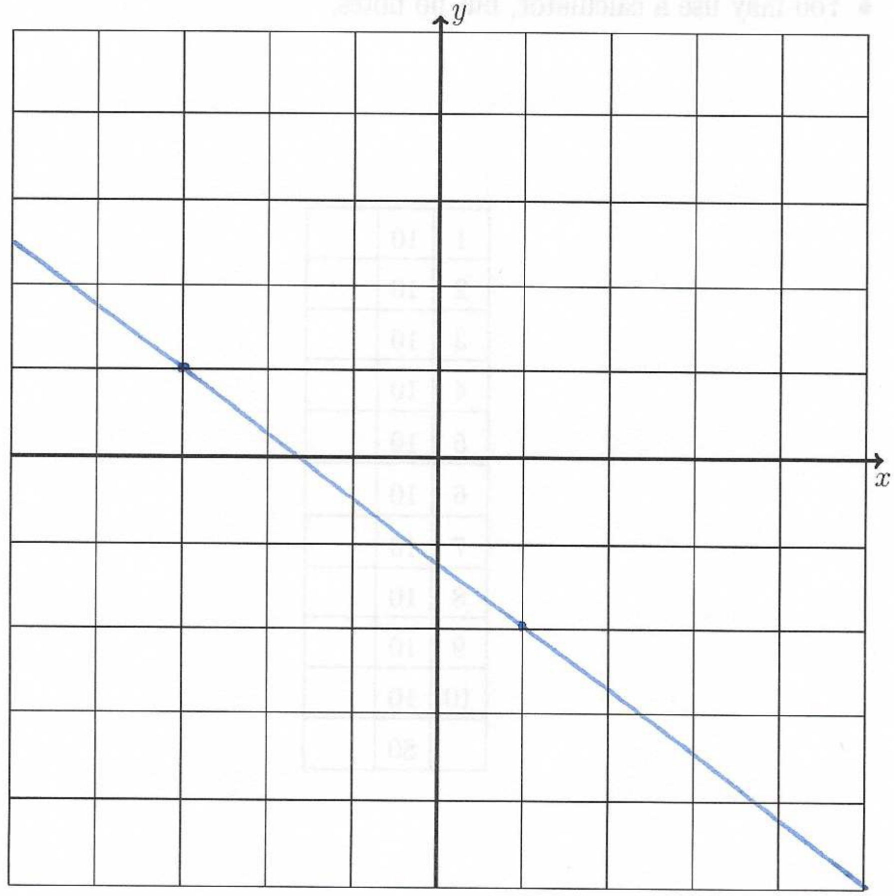
Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-3, 1)$ and $(1, -2)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



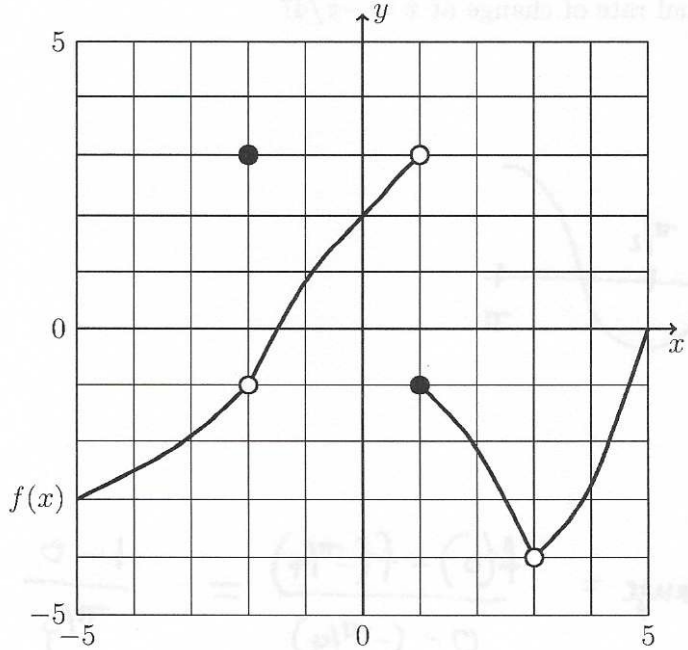
slope $\frac{-2-1}{1-(-3)} = \frac{-3}{4}$

$$\boxed{y-1 = -\frac{3}{4}(x+3)}$$

$$y = -\frac{3}{4}x + 1 - \frac{9}{4}$$

$$y = -\frac{3}{4}x - \frac{5}{4}$$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x\to 3} f(x)$ -4

(b) $\lim_{x\to -2^-} f(x)$ -1

(c) $\lim_{x\to -2^+} f(x)$ -1

(d) $\lim_{x\to -2} f(x)$ -1

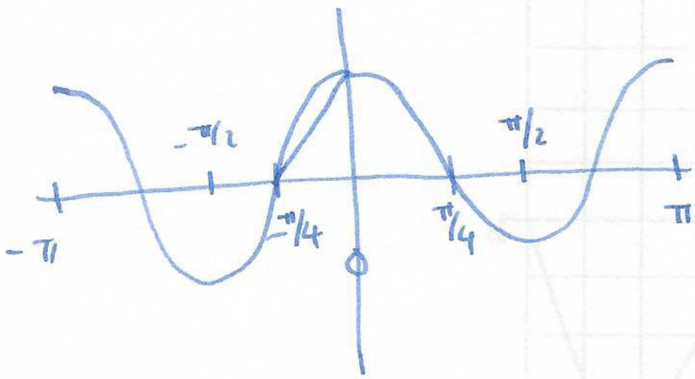
(e) $\lim_{x\to 1^+} f(x)$ -1

(f) $\lim_{x\to 1} f(x)$ DNE

(3) (10 points) Sketch the graph of $f(x) = \cos(2x)$.

(a) What is the average rate of change from $x = -\pi/4$ to $x = 0$?

(b) Looking at the graph, do you expect this to be bigger or smaller than the actual rate of change at $x = -\pi/4$?



a) average rate of change $= \dfrac{f(0) - f(-\pi/4)}{0 - (-\pi/4)} = \dfrac{1-0}{\pi/4} = \dfrac{4}{\pi}$

b) average rate of change on $[-\frac{\pi}{4}, 0]$ smaller than actual rate of change at $-\pi/4$.

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 0} \frac{\sin 4x}{-3x}$$

$4x = \theta$

$$\lim_{\theta \to 0} \frac{\sin\theta}{-3\,\theta/4} = \lim_{\theta\to 0} -\frac{4}{3}\,\frac{\sin\theta}{\theta} = -\frac{4}{3}.$$

(5) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$$

$$\lim_{x \to 2} \frac{(x+4)(x-2)}{(x-2)} = \lim_{x \to 2} x+4 = 6$$

(6) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 2} \frac{x-2}{x-\sqrt{x+2}}$$

$$\lim_{x\to 2} \frac{(x-2)}{(x-\sqrt{x+2})} \frac{(x+\sqrt{x+2})}{(x+\sqrt{x+2})} = \lim_{x\to 2} \frac{(x-2)(x+\sqrt{x+2})}{x^2-x-2}$$

$$= \lim_{x\to 2} \frac{(x-2)(x+\sqrt{x+2})}{(x-2)(x+1)} = \lim_{x\to 2} \frac{x+\sqrt{x+2}}{x+1} = \frac{4}{3}$$

(7) (10 points) Use the limit definition of the derivative to differentiate $f(x) = 2x^2 - x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 - (x+h) - 2x^2 + x}{h} = \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h \cancel{- 2x^2} \cancel{+ x}}{h}$$

$$= \lim_{h \to 0} 4x + 2h - 1 = 4x - 1$$

(8) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{2-x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2-x-h} - \frac{1}{2-x}}{h} = \lim_{h \to 0} \frac{\cancel{2}-\cancel{x} - (\cancel{2}-\cancel{x}-h)}{h(2-x-h)(2-x)}$$

$$= \lim_{h \to 0} \frac{h}{h(2-x-h)(2-x)} = \lim_{h \to 0} \frac{1}{(2-x-h)(2-x)} = \frac{1}{(2-x)^2}$$

(9) (10 points) Find the horizontal asymptotes of $f(x) = \dfrac{\sqrt{x^2+1}}{3x-2}$.

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{3x-2} = \lim_{x\to\infty} \frac{\sqrt{1+1/x^2}}{3-2/x} = \frac{1}{3}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{3x-2} = \lim_{x\to\infty} \frac{\sqrt{1+1/x^2}}{2/x-3} = -\frac{1}{3}$$

(10) (10 points) Sketch the graph of a function for which $f(1) = -2$, f is decreasing for $x < 0$ and increasing for $x > 0$, and $\lim_{x\to\infty} f(x) = 2$.