Sample midterm 1 Solutions

Q1 a) 2 b) −1 c) −3 d) DNE e) 1 f) 2

Q2 a) $\lim_{x\to -1} \frac{3x^2+4x+1}{x+1} = \lim_{x\to -1} \frac{(x+1)(3x+1)}{(x+1)} = \lim_{x\to -1} 3x+1 = -2$

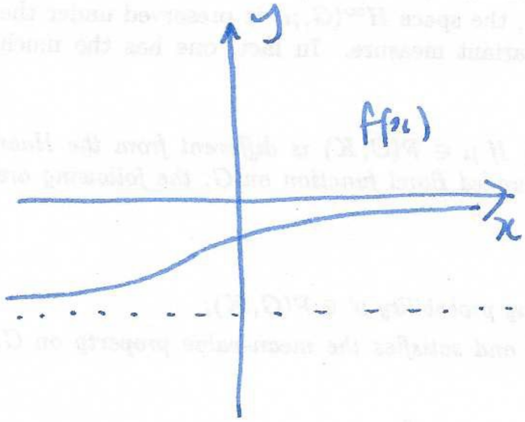
b) $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{\sqrt{x+1}-2}{(\sqrt{x+1}-2)(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$

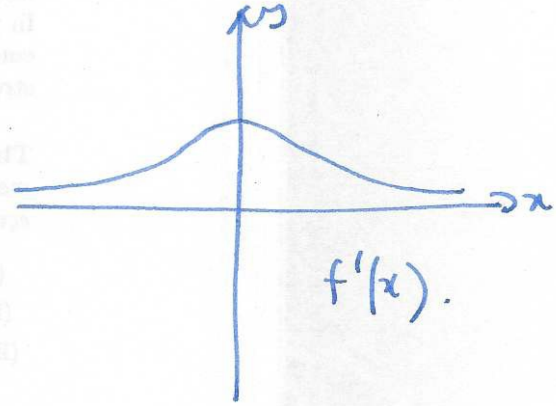
c) $\lim_{x\to 0} \frac{\sin 4x}{\sin 3x} = \lim_{x\to 0} \frac{\sin 4x}{x} \lim_{x\to 0} \frac{x}{\sin 3x}$ $\quad \theta = 4x$, $\phi = 3x$

$= \lim_{x\to 0} \frac{\sin\theta}{\theta/4} \lim_{x\to 0} \frac{\phi/3}{\sin\phi} = 4 \lim_{x\to 0} \frac{\sin\theta}{\theta} \cdot \frac{1}{3} \lim_{x\to 0} \frac{1}{\frac{\sin\phi}{\phi}} = \frac{4}{3}.$

d) $\lim_{x\to 0} \left(\frac{1}{3x} - \frac{1}{x(x+3)}\right) = \lim_{x\to 0} \frac{x+3-3}{3x(x+3)} = \lim_{x\to 0} \frac{1}{3(x+3)} = \frac{1}{9}$

Q3





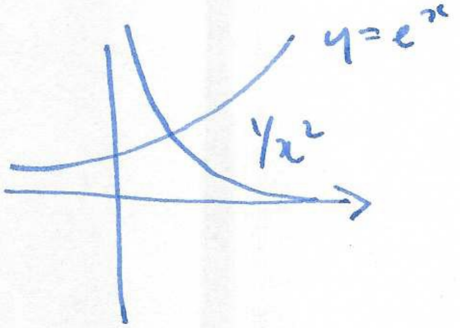
Q4 $\lim_{x\to\infty} \frac{2x^2-1}{\sqrt{6+x^4}} = \lim_{x\to\infty} \frac{2-1/x^2}{\sqrt{1+6/x^4}} = 2$ horizontal asymptote $y = 2$.

$\lim_{x\to -\infty} \frac{2-1/x^2}{\sqrt{1+6/x^4}} = 2.$

Q5 $S = 4\pi r^2$. average rate of change from $r=4$ to $r=5$:

$$\frac{4\pi . 5^2 - 4\pi . 4^2}{5-4} = 4\pi(25-16) = 36\pi .$$

Q6



$e^x - 1/x^2 \longrightarrow -\infty$ as $x \to 0$

$e^x - 1/x^2 > 0$ for $x = 1$ (as $e > 1$)

so IVT $\Rightarrow$ there is an $x \in (0,1)$ with

$e^x = \dfrac{1}{x^2}$.

Q7 $f(x) = \dfrac{1}{\sqrt{x+3}}$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{5+h}} - \frac{1}{\sqrt{5}}}{h} = \lim_{h\to 0} \frac{\sqrt{5} - \sqrt{5+h}}{h\sqrt{5+h}\sqrt{5}}$$

$$= \lim_{h\to 0} \frac{(\sqrt{5}-\sqrt{5+h})(\sqrt{5}+\sqrt{5+h})}{h\sqrt{5}\sqrt{5+h}(\sqrt{5}+\sqrt{5+h})} = \lim_{h\to 0} \frac{5-(5+h)}{h\sqrt{5}\sqrt{5+h}(\sqrt{5}+\sqrt{5+h})} = \lim_{h\to 0}\frac{-h}{h\sqrt{5}\sqrt{5+h}(-)}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{5}\sqrt{5+h}(\sqrt{5}+\sqrt{5+h})} = \frac{-1}{\sqrt{5}.\sqrt{5}(\sqrt{5}+\sqrt{5})} = \frac{-1}{10\sqrt{5}}.$$

Q8 a) $f(x) = -2x^2 + x + 1$

$$f'(x) = \lim_{h\to 0} \frac{-2(x+h)^2 + (x+h) + 1 - (-2x^2+x+1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-2x^2} - 4xh - 2h^2 + \cancel{x} + h + \cancel{1} + \cancel{2x^2} - \cancel{x} - \cancel{1}}{h} = \lim_{h\to 0} -4x - 2h + 1$$

$$= -4x + 1$$

③

b) $f(x) = \dfrac{1}{x-2}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h} = \lim_{h \to 0} \frac{x-2-(x+h-2)}{h(x+h-2)(x-2)}$$

$$= \lim_{h \to 0} \frac{\cancel{x}-\cancel{2}-\cancel{x}-h+\cancel{2}}{h(x+h-2)(x-2)} = \lim_{h \to 0} \frac{-1}{(x+h-2)(x-2)} = \frac{-1}{(x-2)^2}.$$

c) $f(x) = \sqrt{x+2}$

$$f'(x) = \lim_{h \to 0} \frac{\left(\sqrt{x+h+2} - \sqrt{x+2}\right)}{h} \frac{\left(\sqrt{x+h+2} + \sqrt{x+2}\right)}{\left(\sqrt{x+h+2} + \sqrt{x+2}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{x}+h+\cancel{2}-\cancel{x}-\cancel{2}}{h\left(\sqrt{x+h+2} + \sqrt{x+2}\right)} = \lim_{h \to 0} \frac{-1}{\left(\sqrt{x+h+2} + \sqrt{x+2}\right)}$$

$$= \frac{-1}{2\sqrt{x+2}}.$$