Math 231 Calculus 1 Fall 14 Sample Final

(1) Differentiate the following functions. Do not simplify your answers.

(a) $f(x) = \dfrac{\ln(x^2+x)}{x-1}$

(b) $f(x) = e^{-3x}\tan(x+1)$

(c) $f(x) = \sqrt[3]{e^{\sin(x)}+1}$

(2) Evaluate the following integrals.

(a) $\displaystyle\int \frac{(x+1)^2}{\sqrt[3]{x}}\,dx$

(b) $\displaystyle\int_0^{\pi/6} \sin^2(3x)\cos(3x)\,dx$

(c) $\displaystyle\int \frac{1}{9+x^2}\,dx$

(3) Note: the possible answers for limits are a number, $+\infty$, $-\infty$ or "does not exist" (DNE). Justify your answers.

(a) Find $\displaystyle\lim_{x\to 3} \frac{x^2+x-6}{x-3}$.

(b) Find $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{e^x-1}$.

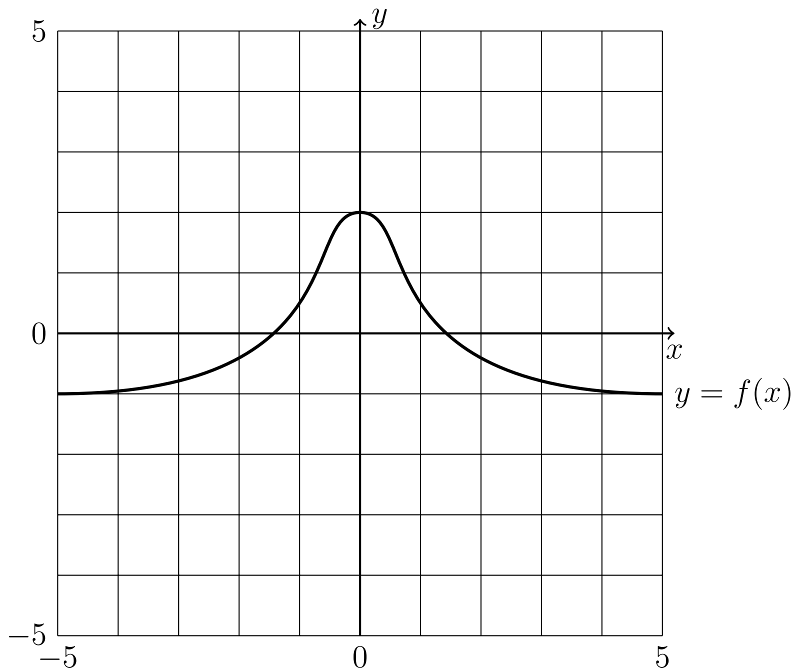
(c) Find $\displaystyle\lim_{x\to 0+} x^{\sin x}$.

(d) Find $\displaystyle\lim_{x\to 0} x\sin(\tfrac{1}{x})$.

(4) Consider $f(x) = x^3 - 3x$.

(a) Find the derivative of $f(x)$, and find the critical points for $f(x)$.
(b) Give the interval(s) for which f is increasing.
(c) Give the intervals for which f is concave up, and for which it is concave down.
(d) Decide which critical points are maxima, minima, or neither.
(e) Sketch the graph of $f(x)$.

(5) Consider the function $f(x)$ defined by the following graph.



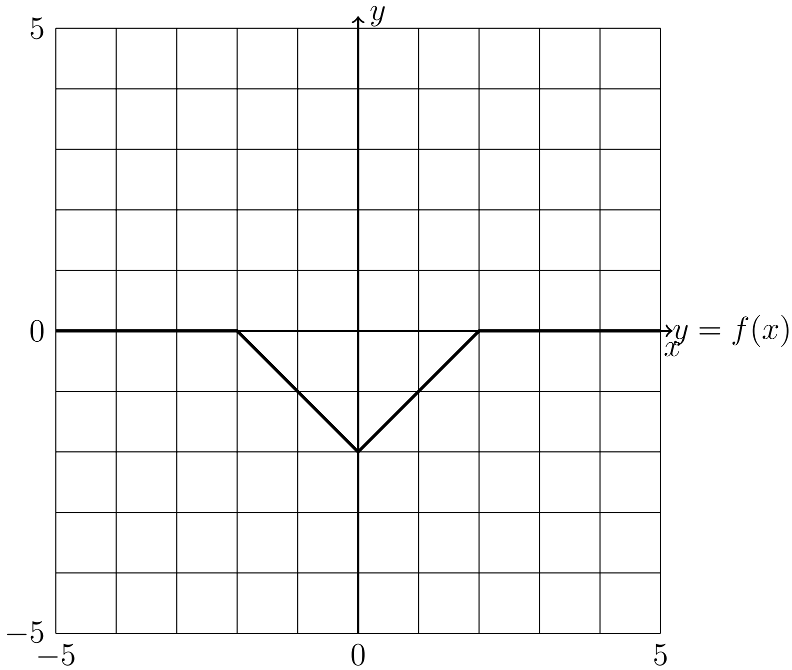
(a) Label all regions where $f(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) Sketch a graph of $f'(x)$ on the figure.

(6) Consider $f(x) = \dfrac{4}{x+2}$.
(a) Sketch the graph of $f(x)$ showing any asymptotes.
(b) Find the slope of the tangent line at $x = 2$, and write down the equation for the tangent line.
(c) Sketch the tangent line at $x = 2$ on your graph.

(7) Let $f(x) = x^2 - 2x$. Find the derivative *using the limit definition of the derivate.* Show all your work.

(8) Use implicit differentiation to find the tangent line to the curve given by the equation $xy^3 + 2x^2y^2 - x^2y = 10$ at the point $(2, -1)$.

(9) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.



(10) A region in the plane is bounded by the x-axis, the graph $y = 25 - x^2$, and the lines $x = -1$ and $x = 2$.

(a) Sketch the region (shading it in) and label the boundaries.

(b) Find the area of the region.

(11) The area of a circular oil slick grows at a rate of 5m^2 per minute. How fast is the area growing when the radius is 12m?

(12) Use linear approximation to estimate $\sqrt[3]{80}$. Use you calculator to find the exact value, and find the absolute and percentage errors.

(13) You wish to build a running track in the shape of a rectangle with two semi-circular ends. If the running rack should have length 400m, what shape minimizes the area?