$\nabla f = \langle 2xy^2, 2x^2y, 2z \rangle$

$\nabla g = \langle 1, 1, 0 \rangle$

$\nabla h = \langle 0, 1, 1 \rangle$

$\nabla f = \lambda \nabla g + \mu \nabla h$

$$\left.\begin{aligned} 2xy^2 &= \lambda \\ 2x^2y &= \lambda + \mu \\ 2z &= \mu \end{aligned}\right\} \; 2x^2y = 2xy^2 - 2z$$

$$\left.\begin{aligned} x+y &= 2 \\ y+z &= 4 \end{aligned}\right\} \; \begin{aligned} y &= 2-x \\ z &= 4-y \end{aligned}$$

$2x^2(2-x) = 2x(2-x)^2 - 2(2+x)$ cubic, find approx solution... $x \approx 0.3$, $y \approx 2.3$, $z \approx 1.7$

§14.1 Integration in many variables

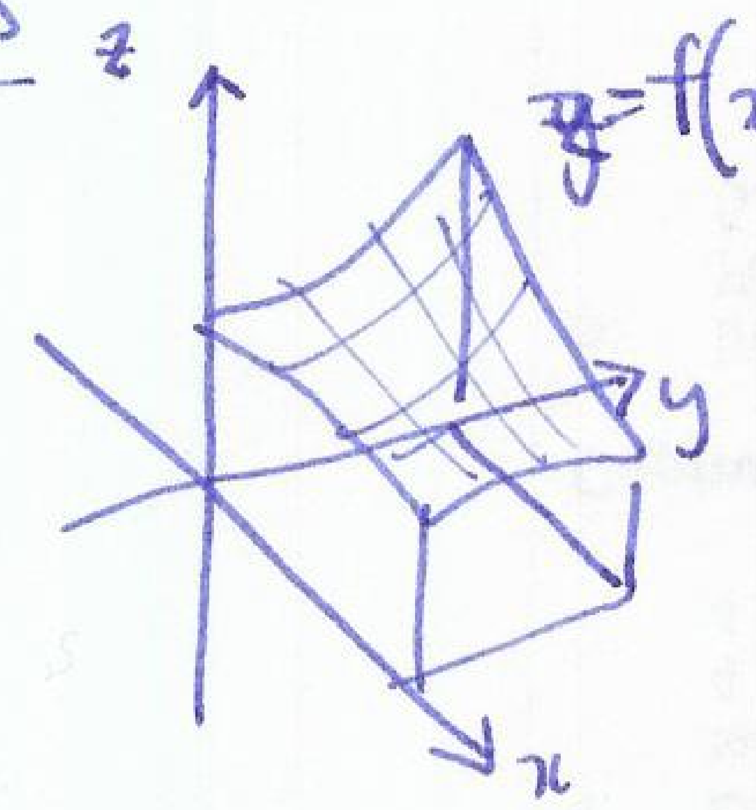
1 var

$y = f(x)$

$\int_a^b f(x)\,dx$ = area under curve between a and b

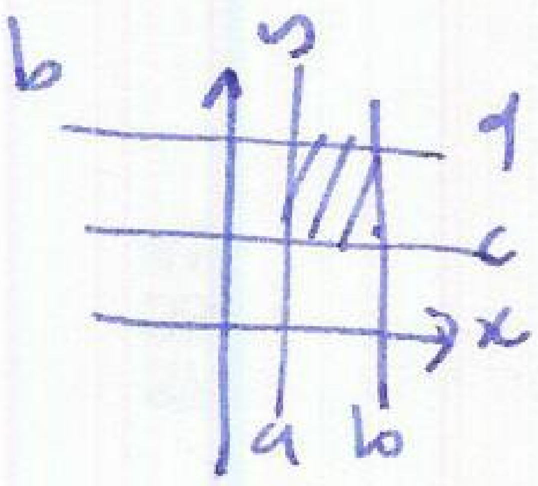
↑ compute this by finding an anti-derivative $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

2 vars



$z = f(x,y)$

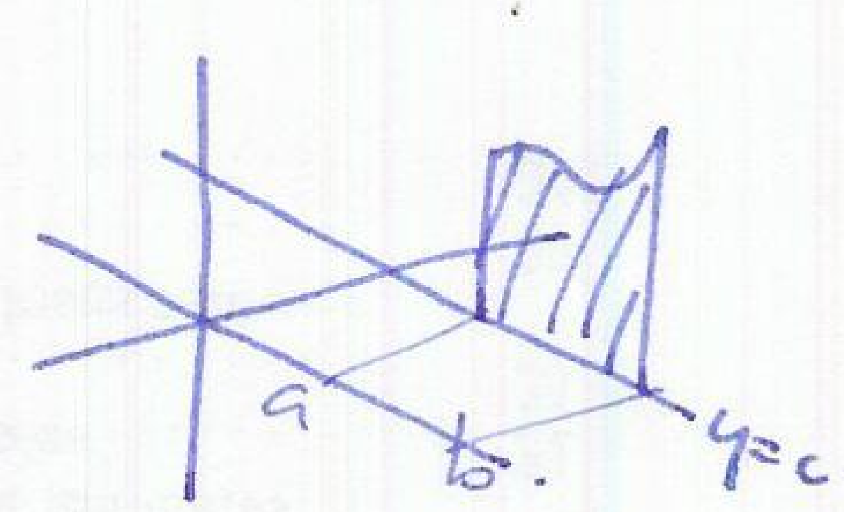
$\int_a^b \int_c^d f(x,y)\,dy\,dx$ = ~~area~~ volume under surface over region $c \le y \le d$, $a \le x \le b$



↑ compute this by doing iterated integrals

recall $\frac{\partial f}{\partial x}$ = "diff wrt x keeping y constant"

$\int_a^b f(x,y)\,dx$ = "integrate wrt x keeping y constant"



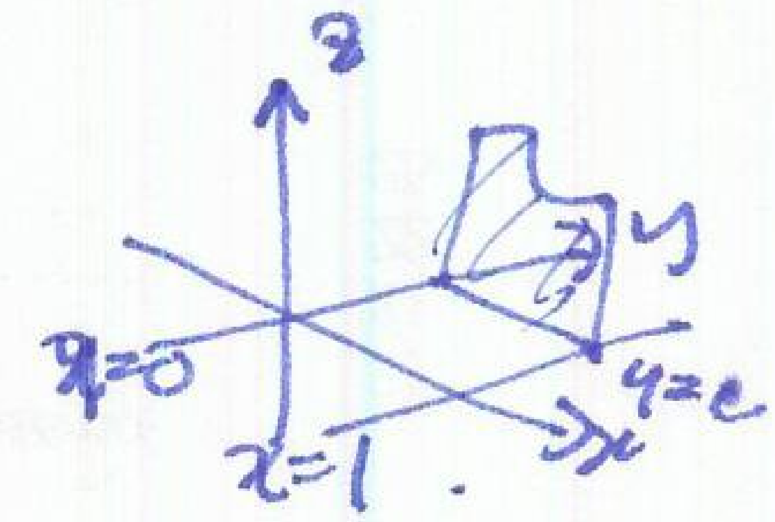
Example $\int_0^1 xy\,dx = \left[\frac{1}{2}x^2y\right]_0^1 = \frac{1}{2}y$

Note: ① There should be no x's remaining!

Warning

② The limits can depend on y!

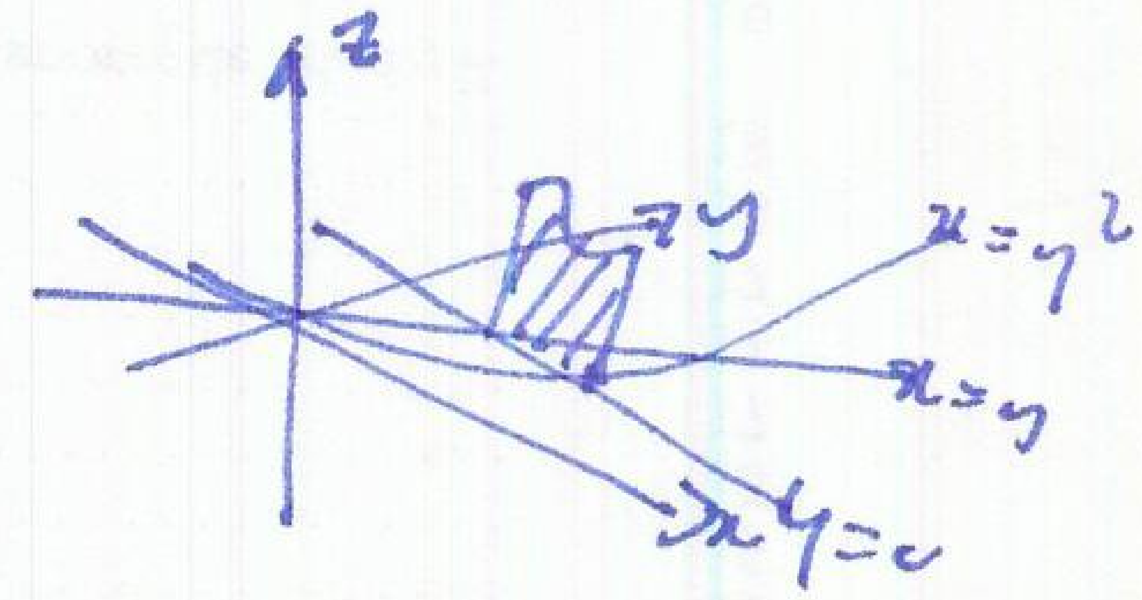
$$\int_y^{y^2} xy\,dx = \left[\tfrac{1}{2}x^2y\right]_y^{y^2} = \tfrac{1}{2}y^5 - \tfrac{1}{2}y^3$$



Warning: can't have x's in the limits!

Q: what does this mean? $\int_0^1 xy\,dy$ means area between 0 and 1

$\int_y^{y^2} xy\,dx$ means area between $x=y$ and $x=y^2$

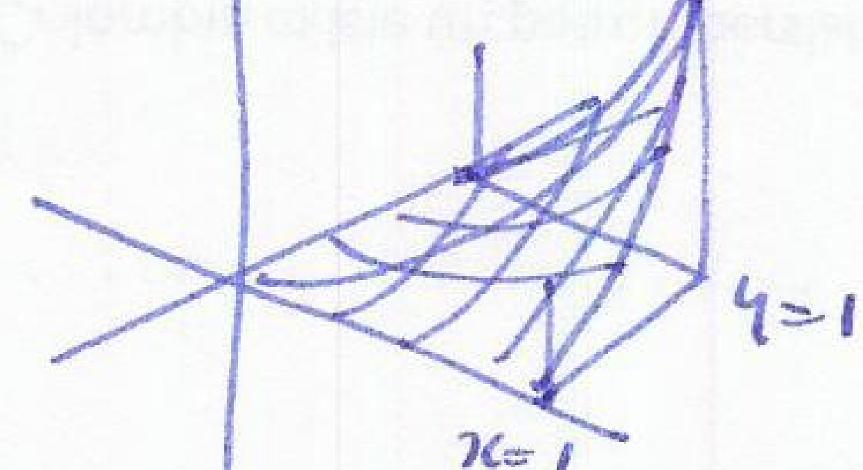


Double integrals

Ex①

$$\int_0^1\int_0^1 xy\,dx\,dy = \int_0^1 \tfrac{1}{2}y\,dy = \left[\tfrac{1}{4}y^2\right]_0^1 = \tfrac{1}{4} = \text{volume above region under surface.}$$

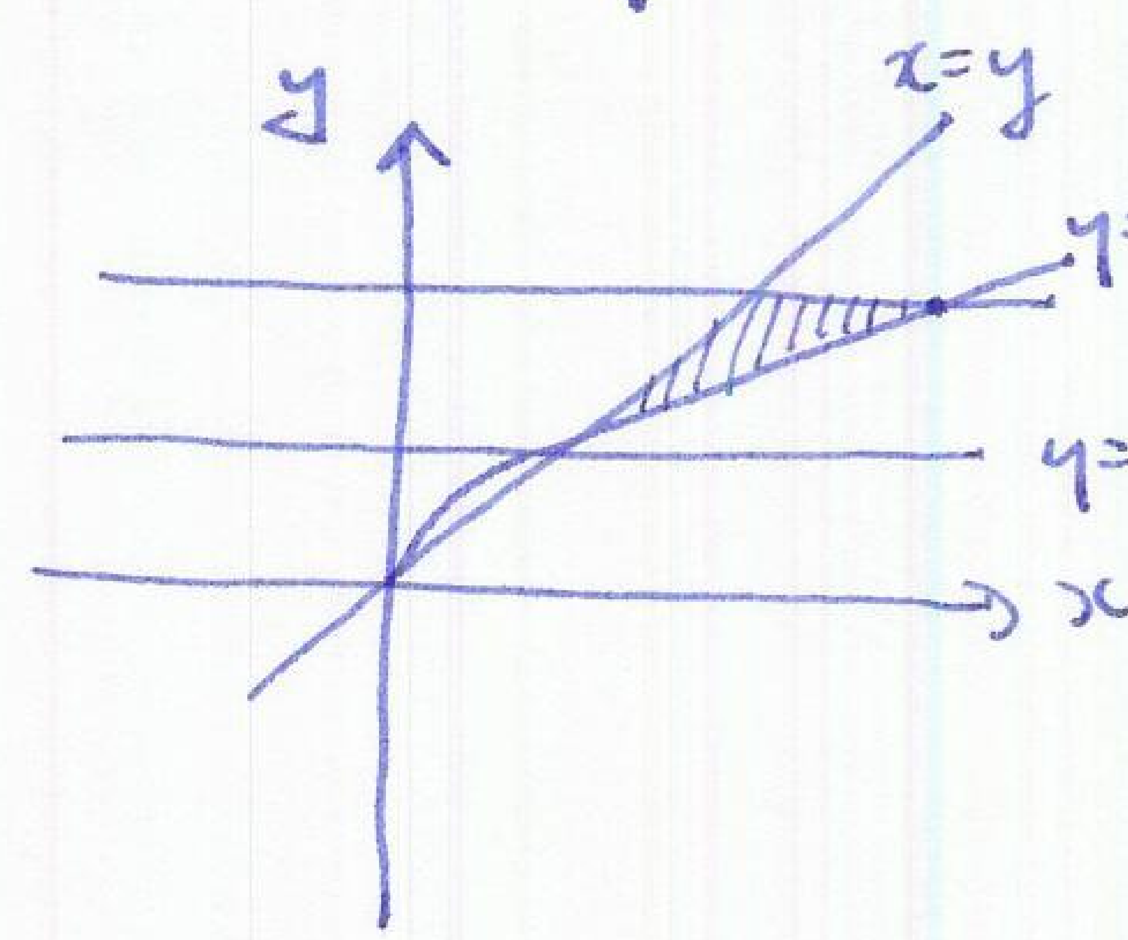
describes region $0 \le x \le 1$, $0 \le y \le 1$



②

$$\int_1^2\int_y^{y^2} xy\,dx\,dy = \int_1^2 \tfrac{1}{2}y^5 - \tfrac{1}{2}y^3\,dy = \left[\tfrac{1}{12}y^6 - \tfrac{1}{8}y^4\right]_1^2 = \text{same number.}$$

limits describe region in xy-plane $1 \le y \le 2$, $y \le x \le y^2$



$x = y^2 \Leftrightarrow y = \sqrt{x}$.

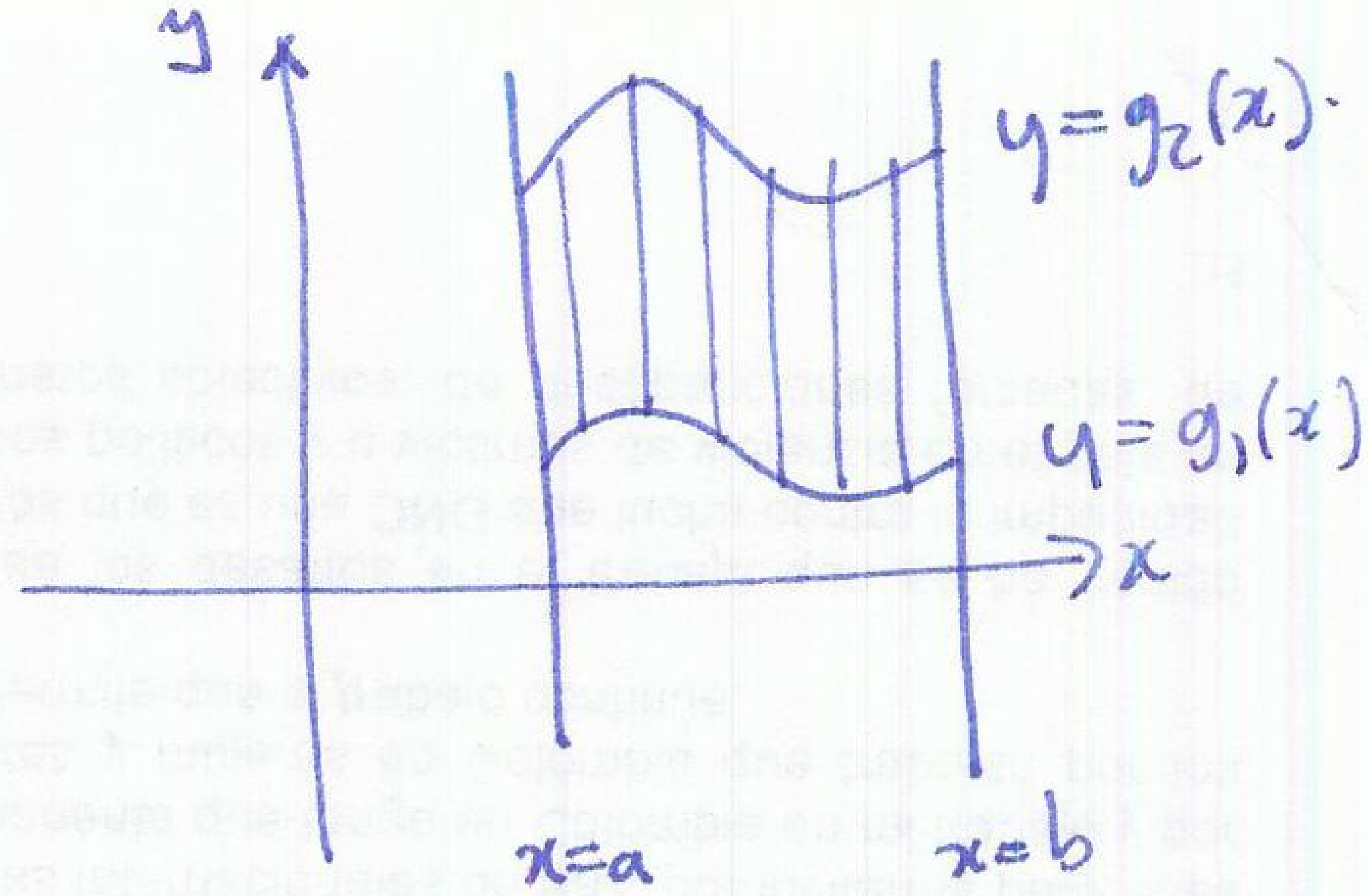
notation $\iint_D f(x,y)\,dx\,dy$

↑ name of region

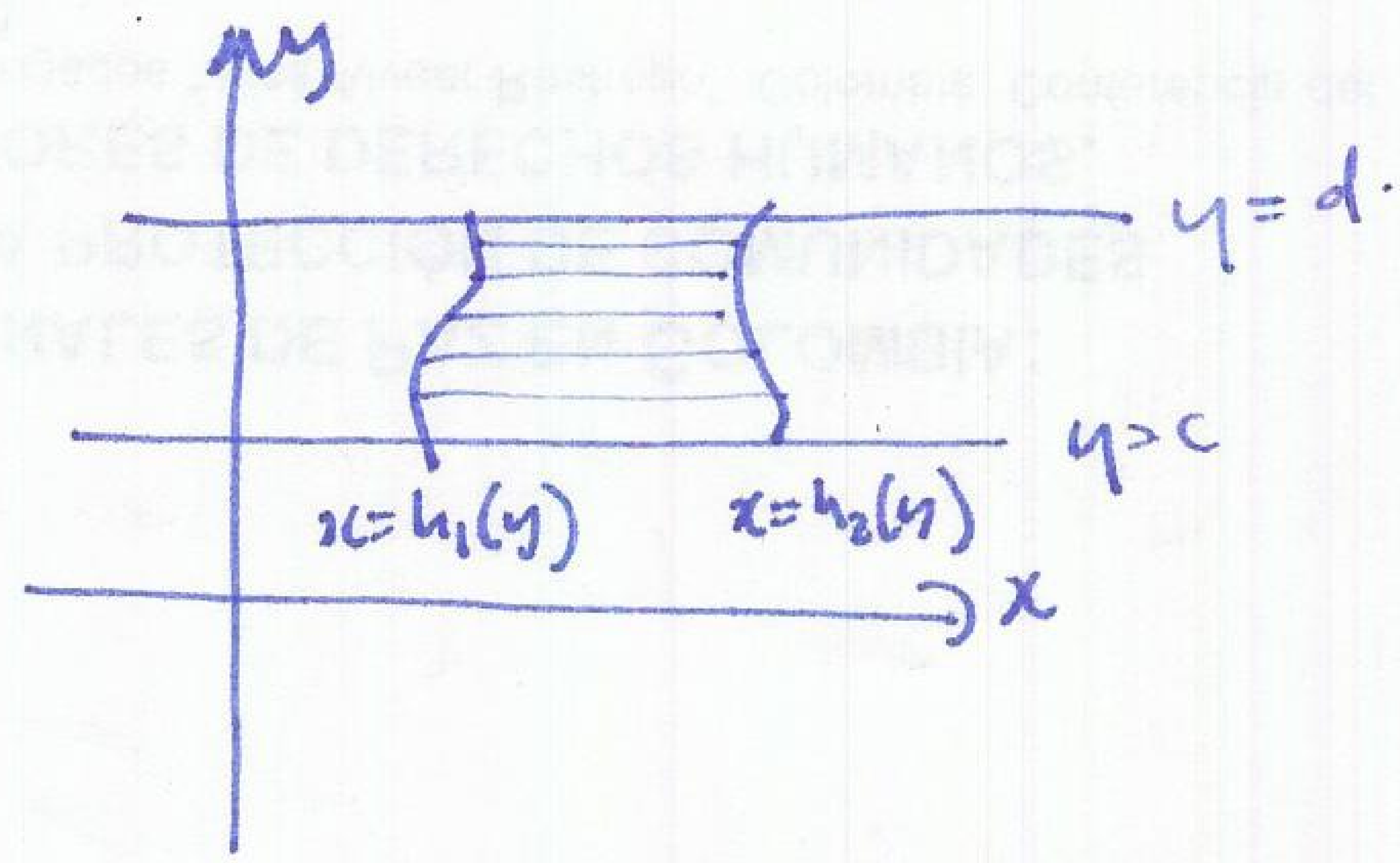
or $\iint_D f(x,y)\,dA$

<u>nice regions</u>

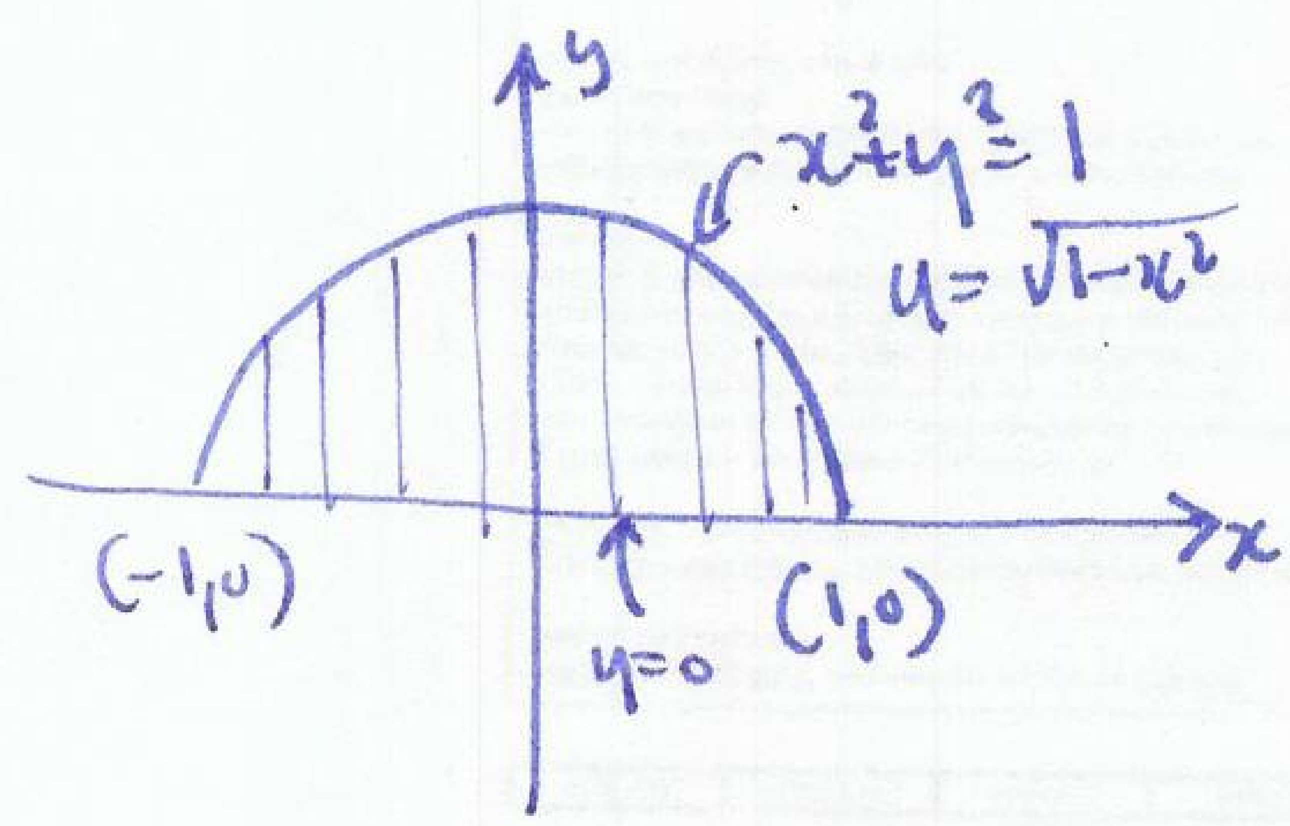
$$\underbrace{\int_a^b}_{\text{must be constants}} \underbrace{\int_{g_1(x)}^{g_2(x)}}_{\text{may be functions of } x} f(x,y)\,dy\,dx$$



$$\underbrace{\int_c^d}_{\text{must be constant}} \underbrace{\int_{h_1(y)}^{h_2(y)}}_{\text{may be functions of } y} f(x,y)\,dx\,dy$$



<u>Example</u> D= upper half disc.



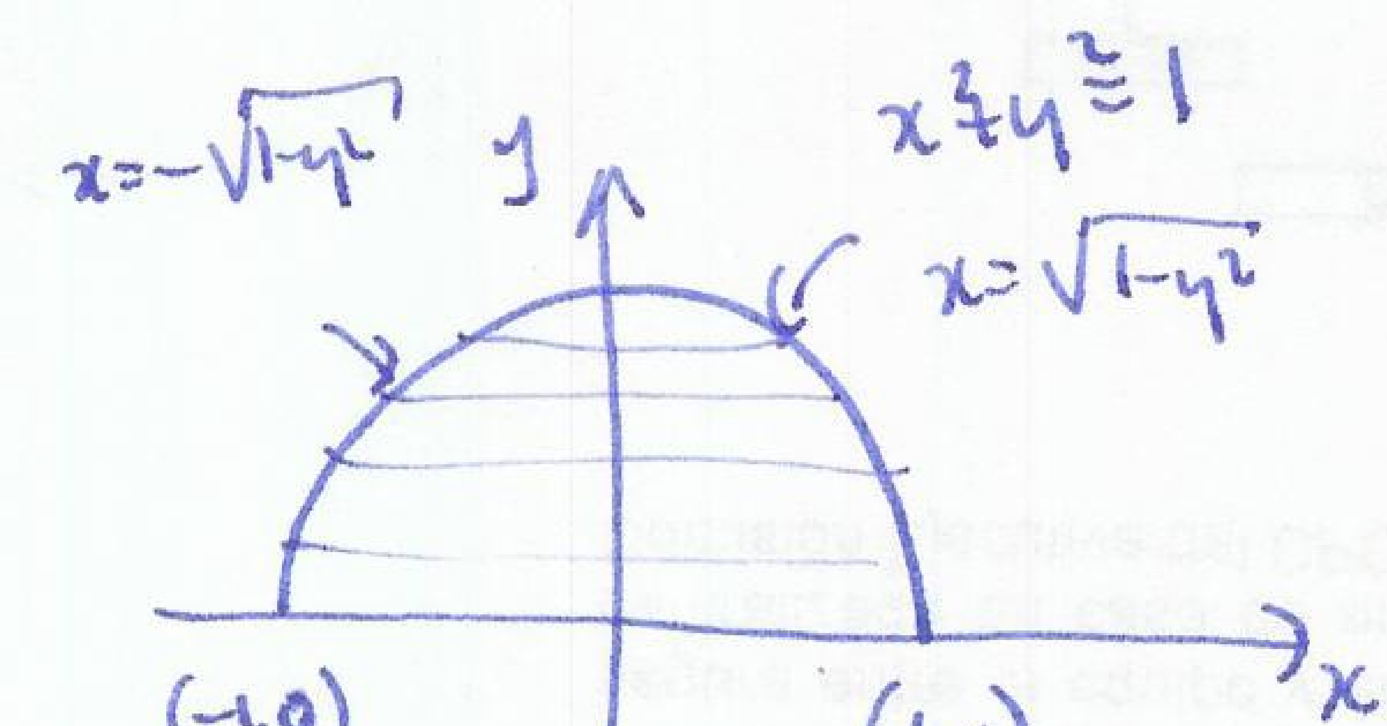
$$\int_{-1}^{+1}\int_{0}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$$

biggest value of x

$\leftarrow$ top boundary

bottom boundary

smallest value of x



$$\int_{0}^{1}\int_{-\sqrt{1-y^2}}^{+\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

biggest value of y

$\leftarrow$ right boundary

left boundary

smallest value of y