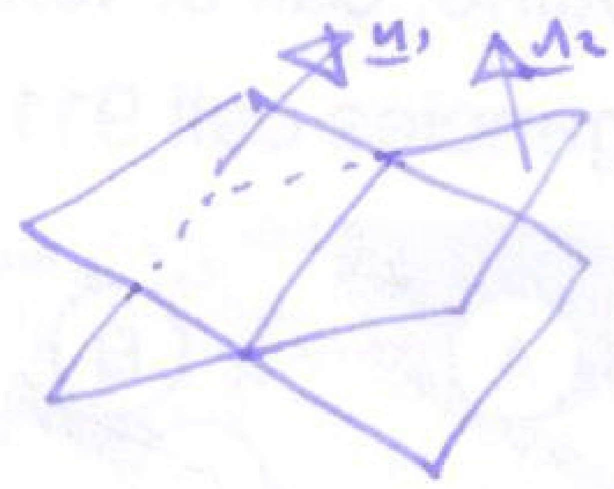
- intersection of two planes



direction vector for line is perpendicular to both $\underline{n}_1, \underline{n}_2$

so $\underline{v} = \underline{n}_1 \times \underline{n}_2$

then just find a point on the line.

Example

$$x+y-z=2 \qquad \underline{n}_1 = \langle 1,1,-1 \rangle$$
$$x+2y+z=4 \qquad \underline{n}_2 = \langle 1,2,1 \rangle$$

$$\underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \underline{i} \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} - \underline{j} \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} + \underline{k} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}.$$
$$= \langle 3, -2, 1 \rangle$$

find point on intersection:

try: set $z=0$: $\left.\begin{array}{l} x+y=2 \quad ① \\ x+2y=4 \quad ② \end{array}\right\}$ solve these ① − ② : $-y=-2$ $y=2$

$x=0$

$z=0$

so $(0,2,0)$ works.

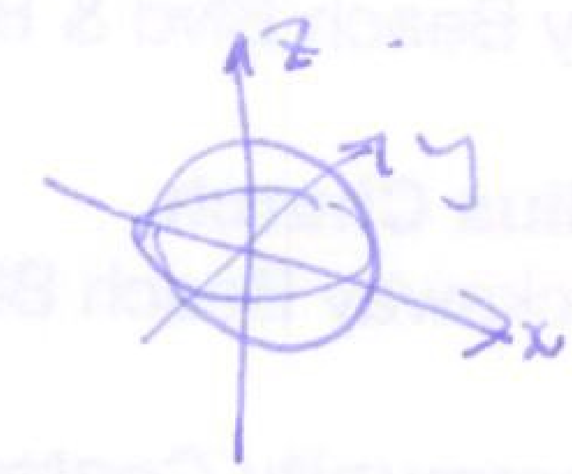
equation: $(\underline{x} - \langle 0,2,0 \rangle) \cdot \langle 3,-2,1 \rangle = 0$.

§12.6 Quadric surfaces

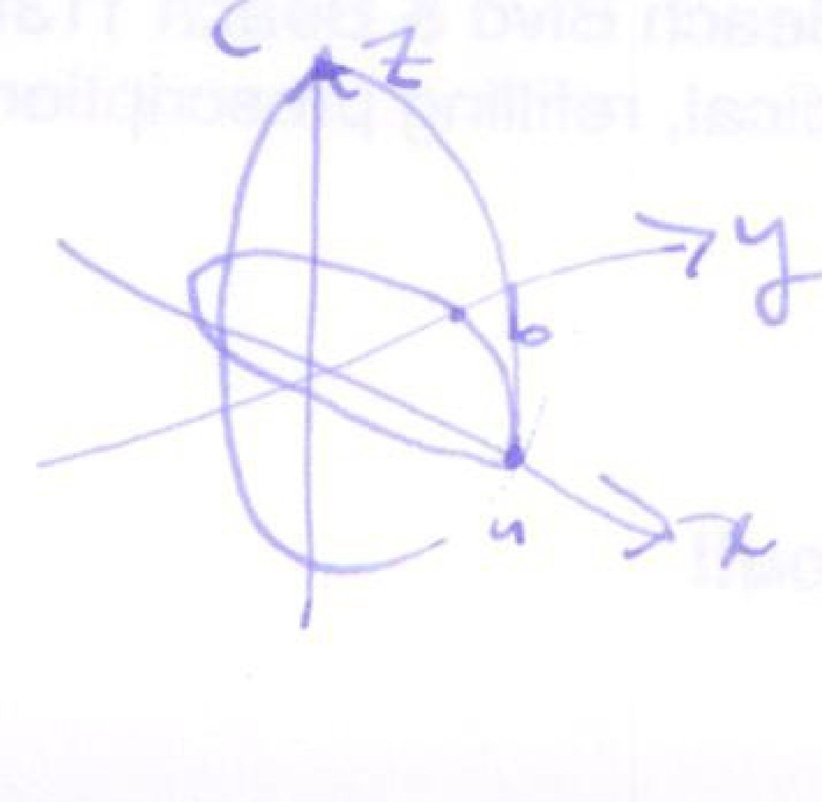
A quadric surface is a surface defined by a quadratic equation in 3 variables.

Examples sphere of radius r:

$$x^2+y^2+z^2=r^2$$



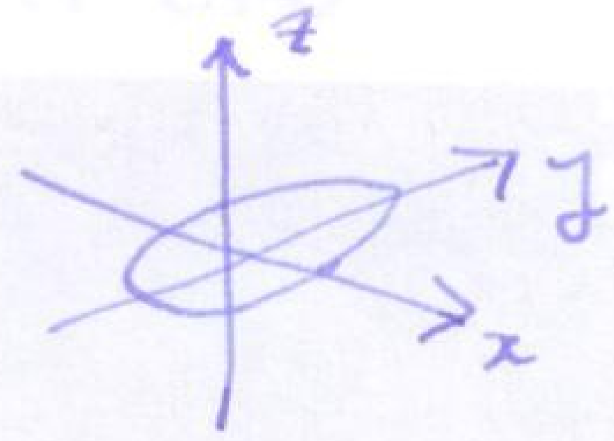
ellipsoids $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$



Q: how do we work out what the surface looks like.

Tool: traces / cross-sections are intersections of the surface with planes parallel to the coordinate axes.

Example $x^2+\frac{y^2}{4}+\frac{z^2}{9}=1$

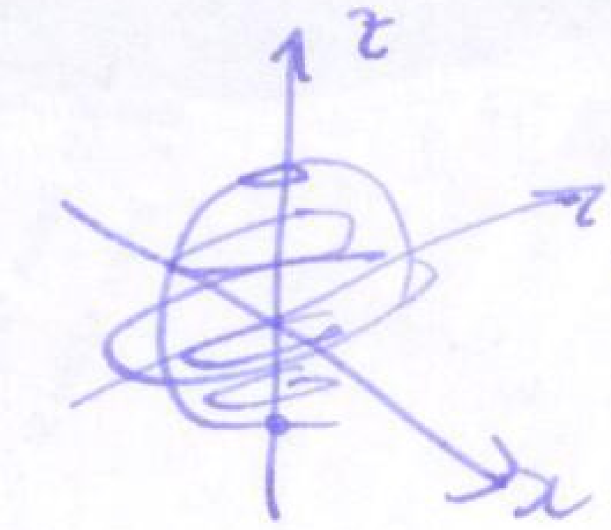


xy-trace (set $z=0$) : $x^2+\frac{y^2}{4}=1$ (ellipse in xy-plane).

what happens when you vary z? $x^2+\frac{y^2}{4}=1-\frac{z^2}{9}$.

z large no solutions ($z>3$)

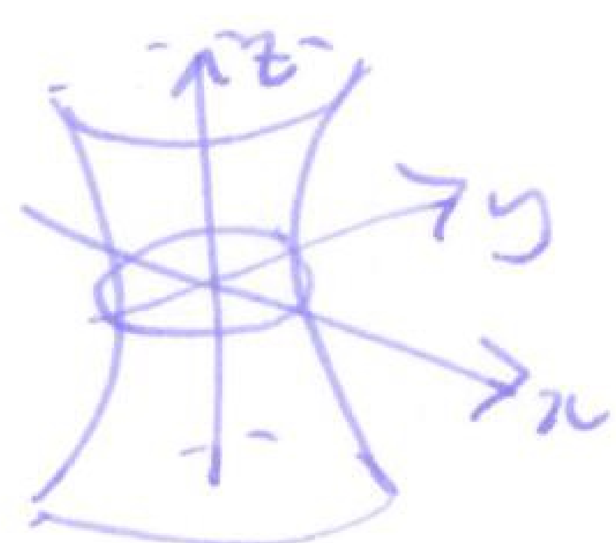
z large negative no solution ($z<-3$)



hyperboloids

$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2-\left(\frac{z}{c}\right)^2=1$

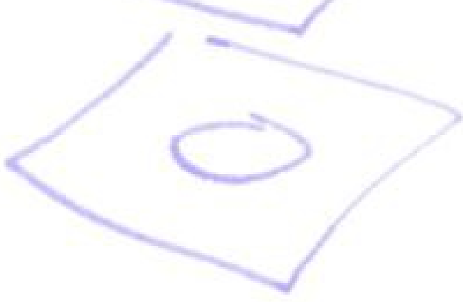
hyperboloid of 1-sheet



traces parallel to xy-plane.

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\underbrace{1+\left(\frac{z}{c}\right)^2}_{\text{always positive, so always solutions}}$$

$z=1$



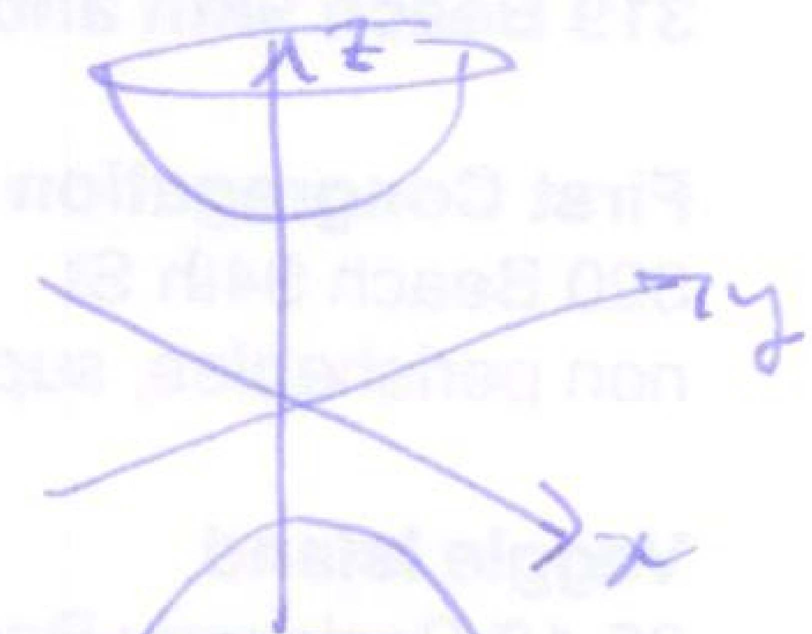
$z=0$

$z=1$

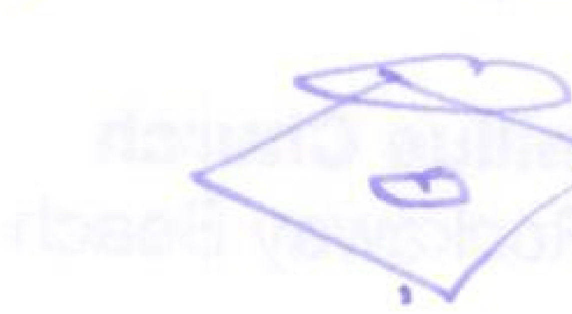
(These hyperbolae are aligned along the z-axis.)

$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2-\left(\frac{z}{c}\right)^2=-1$

hyperboloid of 2-sheets.

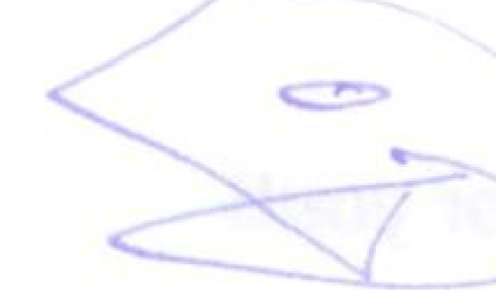


$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=-1+\left(\frac{z}{c}\right)^2.$$



$z>c$ solns

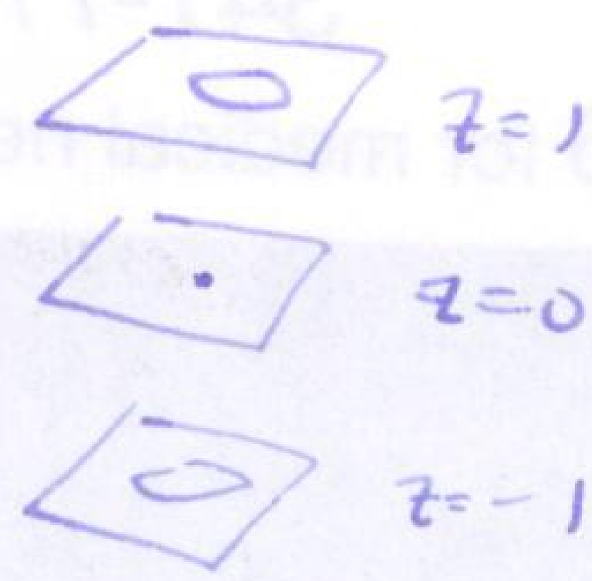
$z=0$ no solution

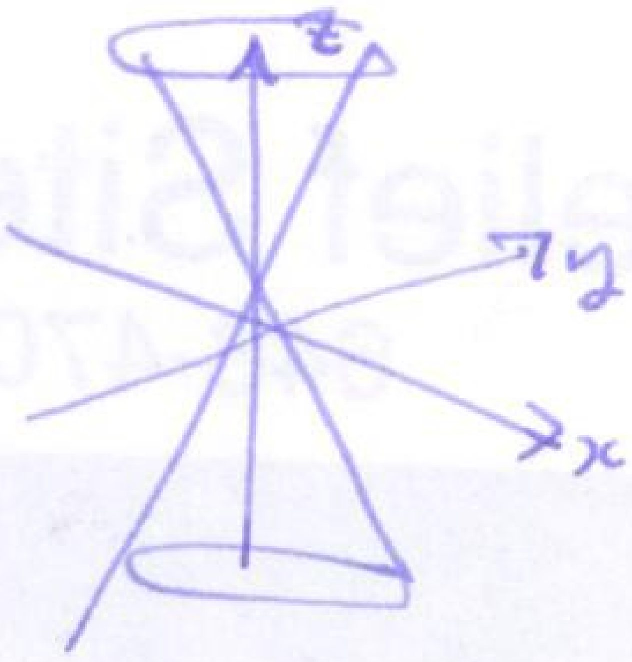


$z<-c$ soln

(elliptic) cones $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=\left(\frac{z}{c}\right)^2$

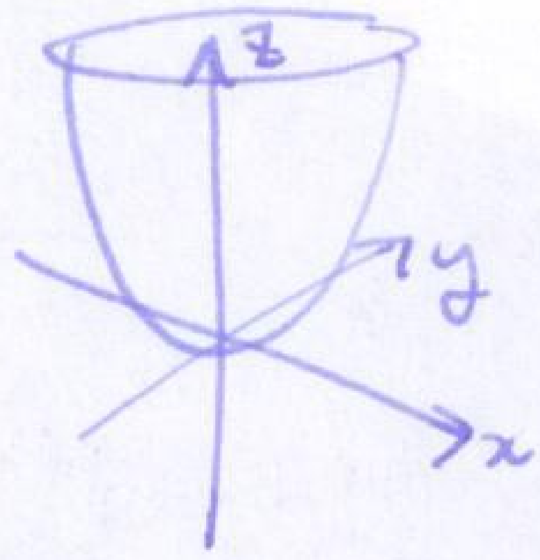
traces / cross-sections parallel to xy-plane





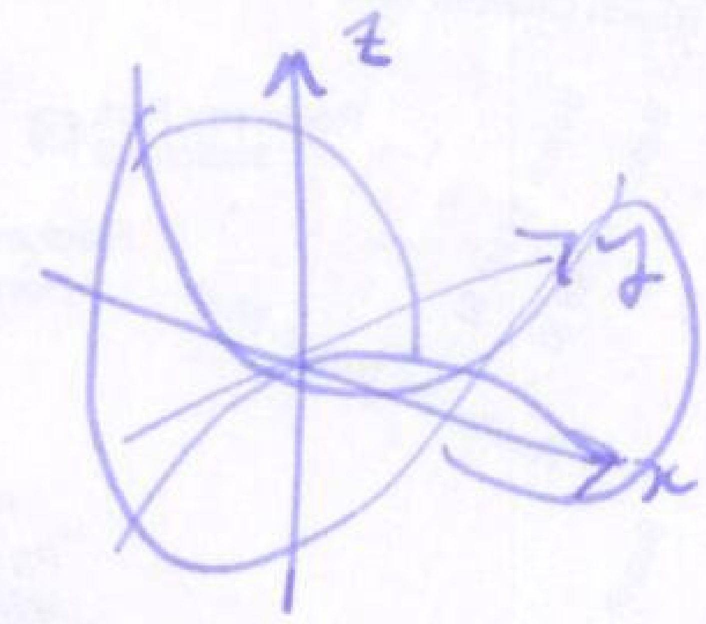
Paraboloid

elliptic paraboloid $z=\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2$

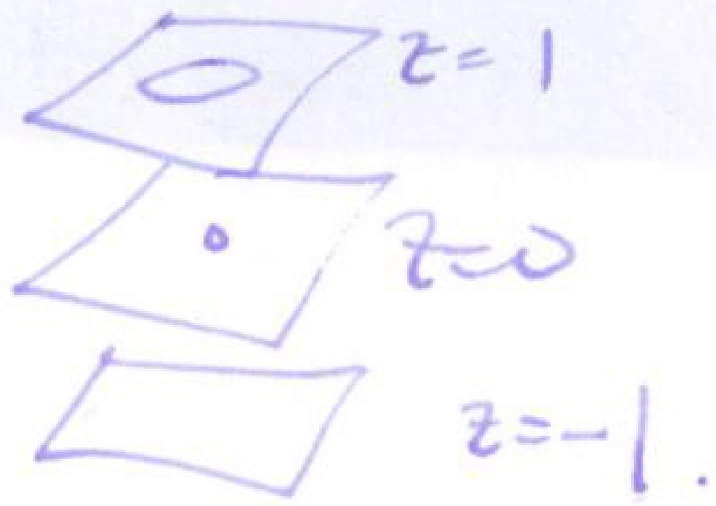


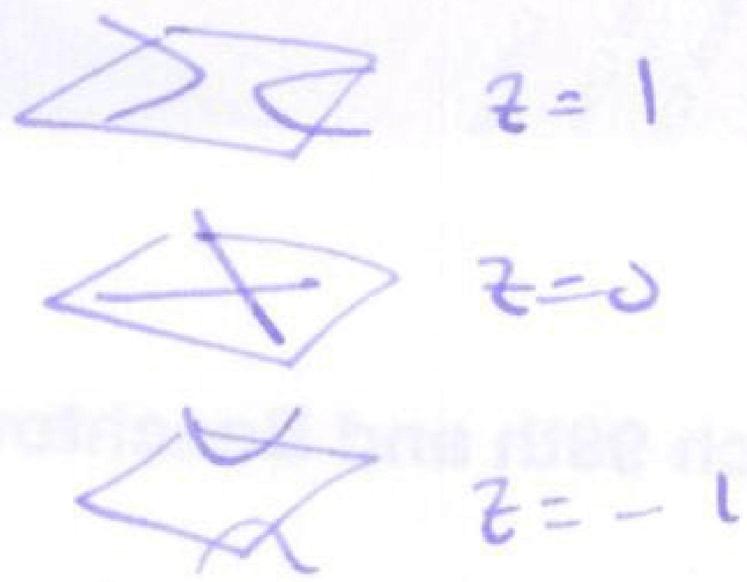
hyperbolic paraboloid $z=\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2$.

(saddle surface)

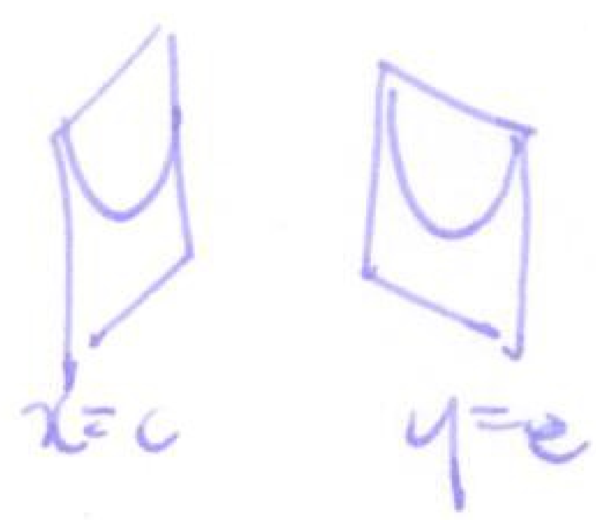


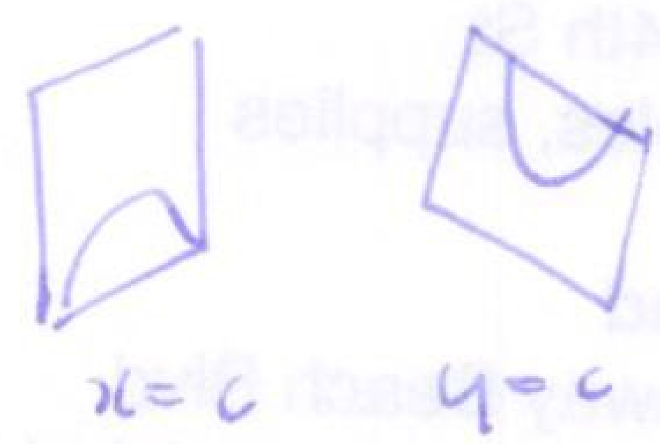
horizontal traces



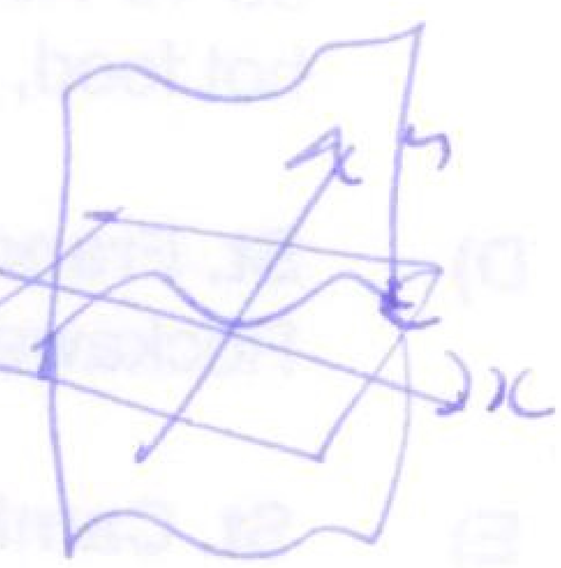


vertical traces



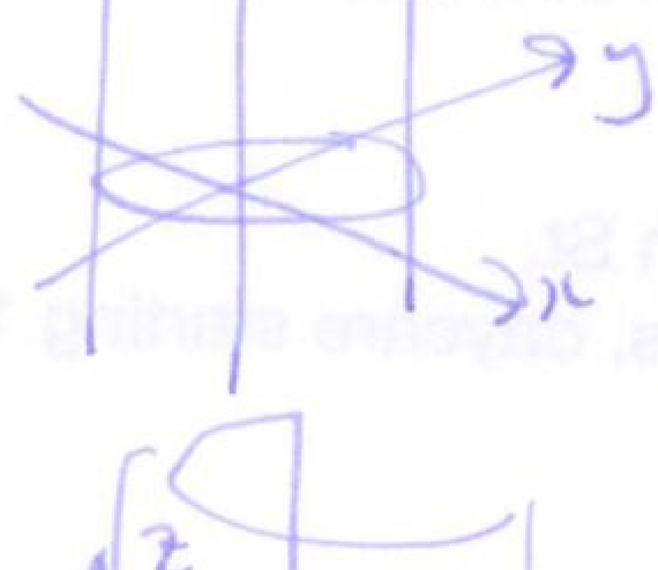


cylinders general cylinder: C some curve in xy-plane:

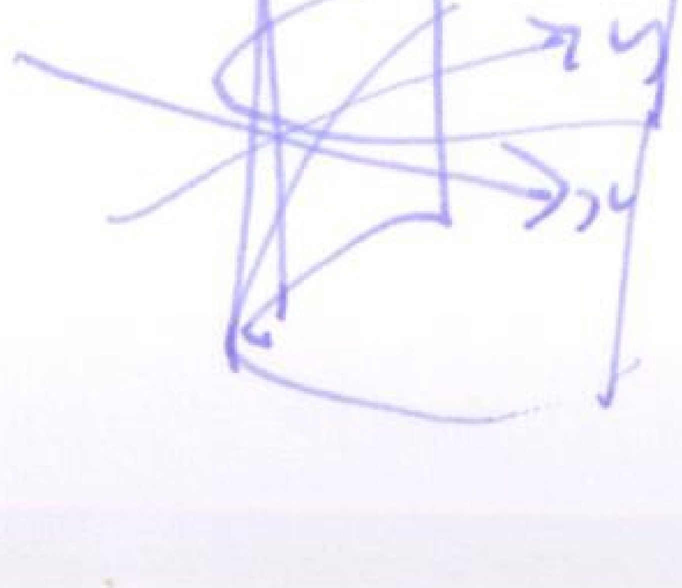


the cylinder over C is all points directly above or below C

Example $x^2+y^2=r^2$



$y=x^2$



§13.1 Vector valued functions

real valued functions of 1-variable $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto f(x)$

paramaterized curves / vector valued functions of 1-variable

$f : \mathbb{R} \to \mathbb{R}^2$ $\qquad\qquad$ $f : \mathbb{R} \to \mathbb{R}^3$

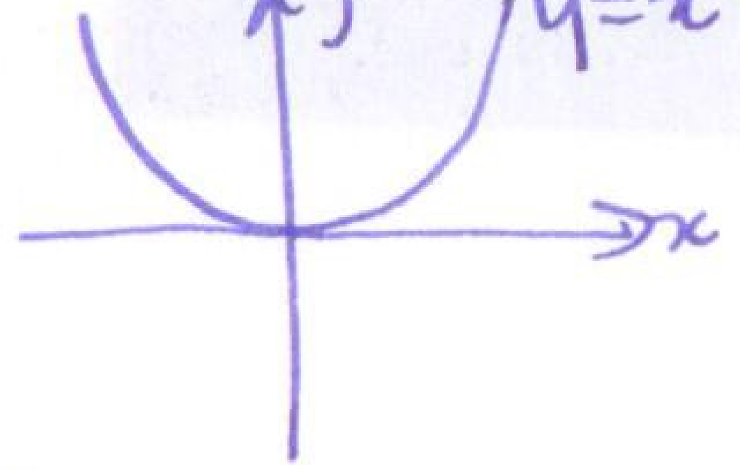
$t \mapsto \langle f(t), g(t) \rangle$ $\qquad\qquad$ $t \mapsto \langle f(t), g(t), h(t) \rangle$

alternate notation: $\underline{r} : \mathbb{R} \to \mathbb{R}^2$

$t \mapsto f(t)\,\underline{i} + g(t)\,\underline{j}$ $\qquad\qquad$ $t \mapsto f(t)\,\underline{i} + g(t)\,\underline{j} + h(t)\,\underline{k}$

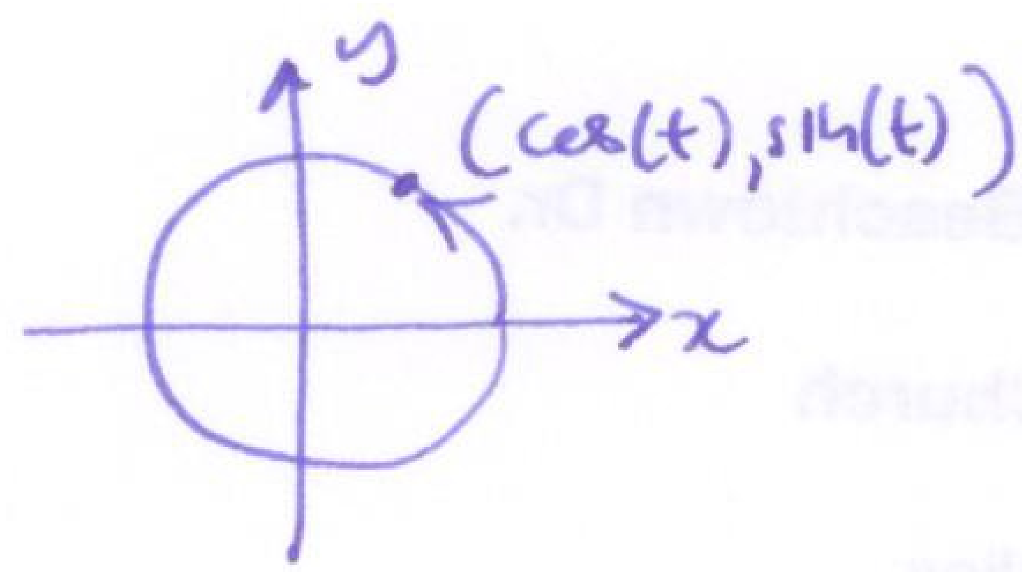
Example

① graph of a function of 1-variable



$x \mapsto \langle x, x^2 \rangle$ $\qquad$ (in general $\langle x, f(x) \rangle$).

② $\underline{r}(t) = \langle \cos(t), \sin(t) \rangle$.



observation $\quad \cos^2(t) + \sin^2(t) = 1$

$\qquad\qquad\quad x^2 + y^2 = 1$

warning: the same curve has many different parameterizations

$\langle \cos t, \sin t \rangle \qquad 0 \le t \le 2\pi$

$\langle \cos(2t), \sin(2t) \rangle \qquad 0 \le t \le \pi$ (twice as fast)

note $\quad \langle \cos(t), \sin(t) \rangle \qquad 0 \le t \le \pi$ only goes half way round

$\qquad \langle \cos(t), \sin(t) \rangle \qquad t \in \mathbb{R}$ goes round infinitely often.

Example $\quad \underline{r}(t) = \langle \cos(t), \sin(t), t \rangle$

(helix)