Math 233 Calculus 3 Spring 13 Sample midterm 3

(1) Write down limits for the following integrals.
   (a) The integral over the region in the octant $x \geq 0, y \leq 0, z \leq 0$ inside the cylinder $x^2 + y^2 = 4$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 4$.
   (b) The integral over region with $y \leq 0$, which lies below the negative cone $z^2 = 3x^2 + 3y^2$ with $z \leq 0$, and inside the sphere of radius 5.
   (c) The integral over the tetrahedron with vertices $(0,0,0)$, $(0,1,0)$, $(0,1,1)$ and $(1,1,1)$.

(2) Find the volume of the solid contained in the cylinder $x^2 + y^2 = 4$, below the surface $z = (x+y)^2$ and above the surface $z = -2(x-y)^2$.

(3) Use spherical coordinates to evaluate the following integral.

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}} dz dy dx$$

(4) Evaluate

$$\int \int_R (y-x)^2 dA,$$

where R is the region bounded by the lines $y = x$, $y = 2x$, $y = x+2$ and $y = 2x - 1$, using the change of variable $T(u,v) = (u - v, 2u - v)$.

(5) Let $f(x,y,z) = e^x + zy$. Evaluate

$$\int_C f ds,$$

where C is the straight line path from $(-1,3,2)$ to $(4,6,6)$.

(6) Show that the vector field $\mathbf{F} = \langle y^2, x, -z \rangle$ is not conservative. Evaluate

$$\int_C \mathbf{F}.d\mathbf{s}$$

where C is the circle of radius 4 in the plane $z = 1$ centered on the z-axis.

(7) Show that the vector field $\mathbf{F} = \langle ze^{xz}, -z\sin(yz), xe^{xz} - y\sin(yz)\rangle$ is conservative, and find a function $f(x, y, z)$ such that $\nabla f = \mathbf{F}$. Evaluate

$$\int_C \mathbf{F}.d\mathbf{s}$$

where C is the curve formed by the intersection of the plane $z = 2x + 3y$ with the sphere of radius 36 in the positive octant, oriented anticlockwise around the z-axis.

(8) Find the surface area of the paraboloid $z = 16 - x^2 - y^2$ in the first octant.

(9) Find the integral of the vector field $\mathbf{F} = \langle x, y, x + y\rangle$ over the surface on the paraboloid $z = x^2 + y^2$ lying over the unit disc in the xy-plane.

(10) Use Green's Theorem to evaluate $\int_C \mathbf{F}d\mathbf{s}$, where $\mathbf{F} = \langle x + y, x^2 - y\rangle$ and C is the boundary of the region enclosed by $y = x^2$ and $y = \sqrt{x}$ for $0 \leq x \leq 1$.

(11) Use Stokes' Theorem to evaluate the integral of curl(F) through the part of the cone $z^2 = x^2 + y^2$, with $2 \leq z \leq 4$, and with the outward pointing normal, where $\mathbf{F} = \langle x^2 + y^2, x + z^2, 0\rangle$.