Math 233 Calculus 3 Spring 13 Midterm 2b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the tangent plane to the surface $z = xy - xy^2 + 4$ at the point $(1, -2, -2)$.

$z = L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\frac{\partial f}{\partial x} = y - y^2$ $\quad f_x(1,-2) = -6$

$\frac{\partial f}{\partial y} = x - 2xy$ $\quad f_y(1,-2) = 5$

$z = -2 - 6(x-1) + 5(y+2)$

(2) (10 points) Find the linear approximation to the function $f(x, y, z) = e^{xy} + \tan^{-1}(3y + 4z)$ at the point $(1, 2, -1)$.

$L(x,y,z) = f(a,b,c) + f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$

$f_x = ye^{xy} + \cancel{\frac{1}{1+(3y+4z)^2}}$ $\qquad f_x(1,2,-1) = 2e^2$

$f_y = xe^{xy} + \frac{1}{1+(3y+4z)^2} \cdot 3$ $\qquad f_y(1,2,-1) = e^2 + \frac{3}{5}$

$f_z = \frac{1}{1+(3y+4z)^2} \cdot 4$ $\qquad f_z(1,2,-1) = \frac{4}{5}$

$L(x,y,z) = e^2 + \tan^{-1}(2) + 2e^2(x-1) + (e^2 + \frac{3}{5})(y-2) + \frac{4}{5}(z+1)$

(3) (10 points) Find the gradient vector for the function $f(x, y, z) = x^2 + y^2 - z$ at the point $(1, 2, 4)$. Use this to find the tangent plane to the surface $z = x^2 + y^2 + 1$ at the point $(1, 2, 4)$.

$\nabla f = \langle 2x, 2y, -1 \rangle$

$\nabla f(1,2,4) = \langle 2, 4, -1 \rangle$

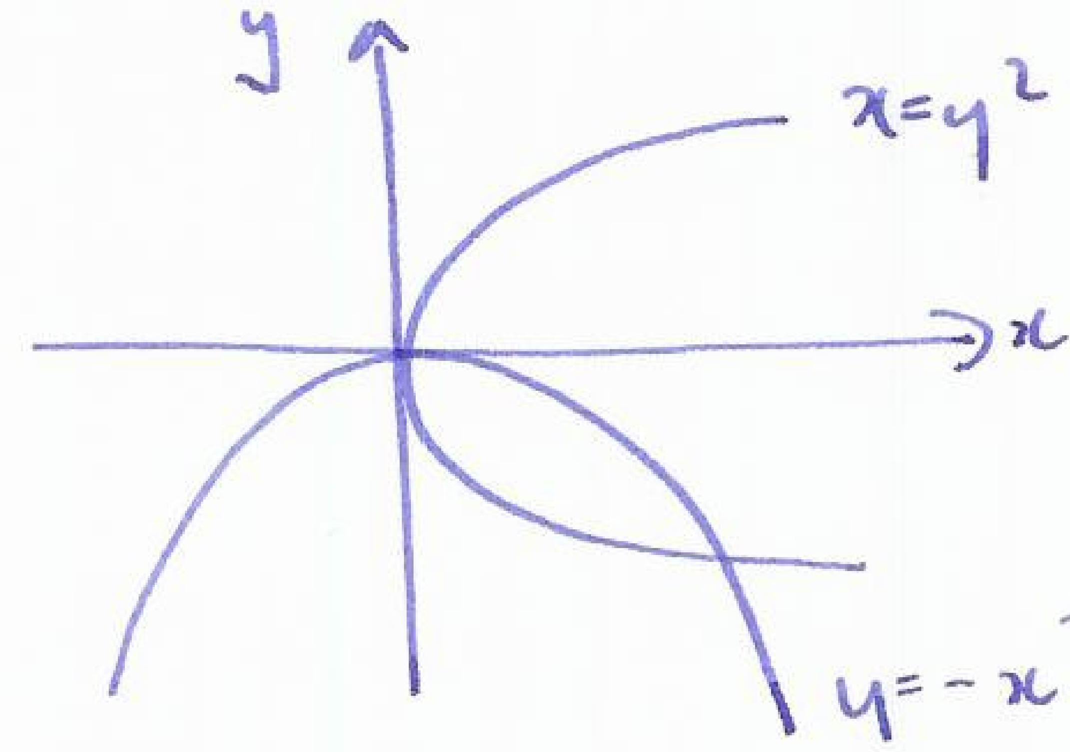
normal to tangent plane

$2x + 4y - z = d$ through $(1,2,4)$

$2 + 8 - 4 = d = 6$

$z = -6 + 2x + 4y$

(4) (10 points) Find the critical points of $f(x, y) = y^3 - x^3 - 3xy$, and use the second derivative test to classify them, if possible.



$f_x = -3x^2 - 3y = 0 \; : \; y = -x^2$

$f_y = 3y^2 - 3x = 0 \; : \; x = y^2$

critical points $(0,0)$, $(1,-1)$

$f_{xx} = -6x$ $\qquad$ $D(0,0) = 0 - (-3)^2 = -9 < 0$ saddle

$f_{yy} = 6y$ $\qquad$ $D(1,-1) = (-6)(-6) - (-3)^2 = 27 > 0$, $f_{xx} < 0$ } maximum

$f_{xy} = -3$

(5) (10 points) Find the maximum and minimum values of $x - y$ on the disc $x^2 + y^2 \leq 9$.

$f(x,y) = x - y$

$g(x,y) = x^2 + y^2 = 9$

$\left.\begin{array}{l} f_x = 1 \\ f_y = -1 \end{array}\right\}$ no critical points

$\nabla f = \lambda \nabla g$

$\left.\begin{array}{l} \left.\begin{array}{l} 1 = \lambda 2x \\ -1 = \lambda 2y \end{array}\right\} x = -y \\ x^2 + y^2 = 9 \end{array}\right\}$

$2x^2 = 9 \qquad x = \pm \frac{3}{\sqrt{2}}$

$\left(\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right) \quad \left(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

$f\left(\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right) = \frac{6}{\sqrt{2}}$ max

$f\left(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right) = -\frac{6}{\sqrt{2}}$ min

(6) (10 points) Use Lagrange's method to find the closest point on the origin to the plane $3x + 2y + z = 6$.

min $f(x,y,z) = x^2+y^2+z^2$ subject to $g(x,y,z) = 3x+2y+z = 6$

$\nabla f = \langle 2x, 2y, 2z \rangle$ $\quad \nabla g = \langle 3, 2, 1 \rangle$

$\nabla f = \lambda \nabla g$

$$2x = 3\lambda$$
$$2y = 2\lambda$$
$$2z = \lambda$$

$3x+2y+z = 6$ $\quad \frac{9\lambda}{2} + 2\lambda + \frac{\lambda}{2} = 6$ $\quad 7\lambda = 6$

$\lambda = 6/7$

$\left(\frac{9}{7}, \frac{6}{7}, \frac{3}{7}\right)$

(7) (10 points) Evaluate

$$\int_0^2 \int_y^2 e^{2x+3y}\,dx\,dy$$

$$\left[\frac{1}{2}e^{2x+3y}\right]_y^2 = \frac{1}{2}e^{4+3y} - \frac{1}{2}e^{5y}$$

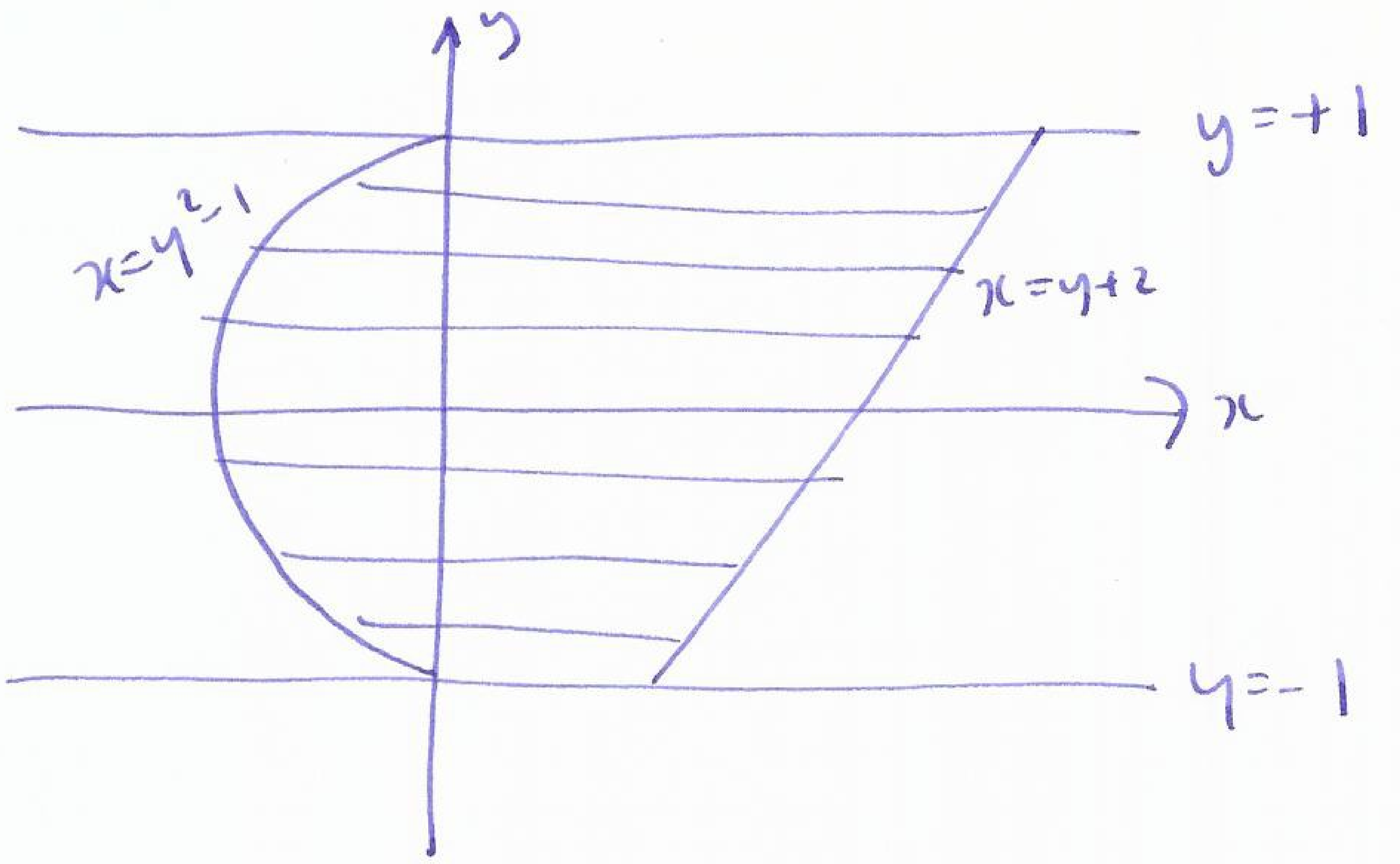
$$\left[\frac{1}{6}e^{4+3y} - \frac{1}{10}e^{5y}\right]_0^2 = \frac{1}{6}e^{10} - \frac{1}{10}e^{10} - \frac{1}{6}e^{4} - \frac{1}{10}$$

(8) Draw the region in the plane corresponding to the limits of the following double integral.

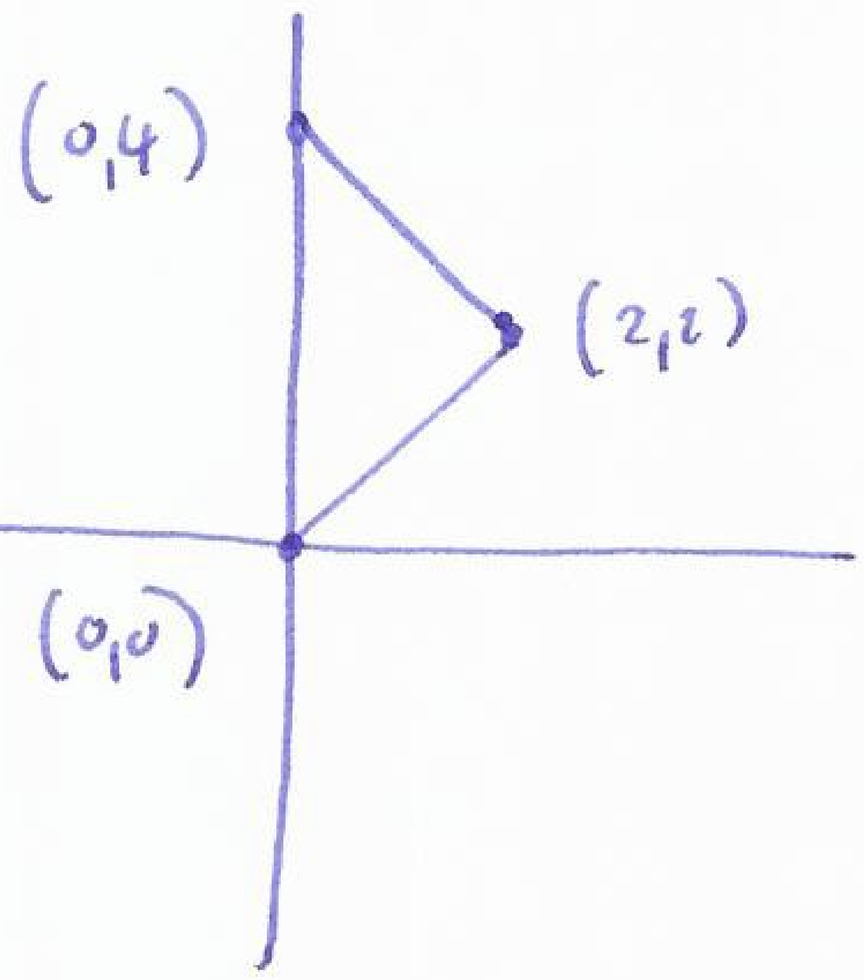
$$\int_{-1}^{1}\int_{y^2-1}^{y+2} f(x,y)\,dx\,dy$$

$x = y+2$

$x = y^2 - 1$



(9) Find the integral of the function $f(x, y) = y - x$ over the triangle with vertices $(0, 0)$, $(0, 4)$ and $(2, 2)$.



$$\int_0^2 \int_x^{4-x} y - x \; dy \, dx$$

$$\left[\frac{1}{2} y^2 - xy \right]_x^{4-x} = \frac{1}{2}(4-x)^2 - x(4-x) - \frac{1}{2}x^2 + x^2$$

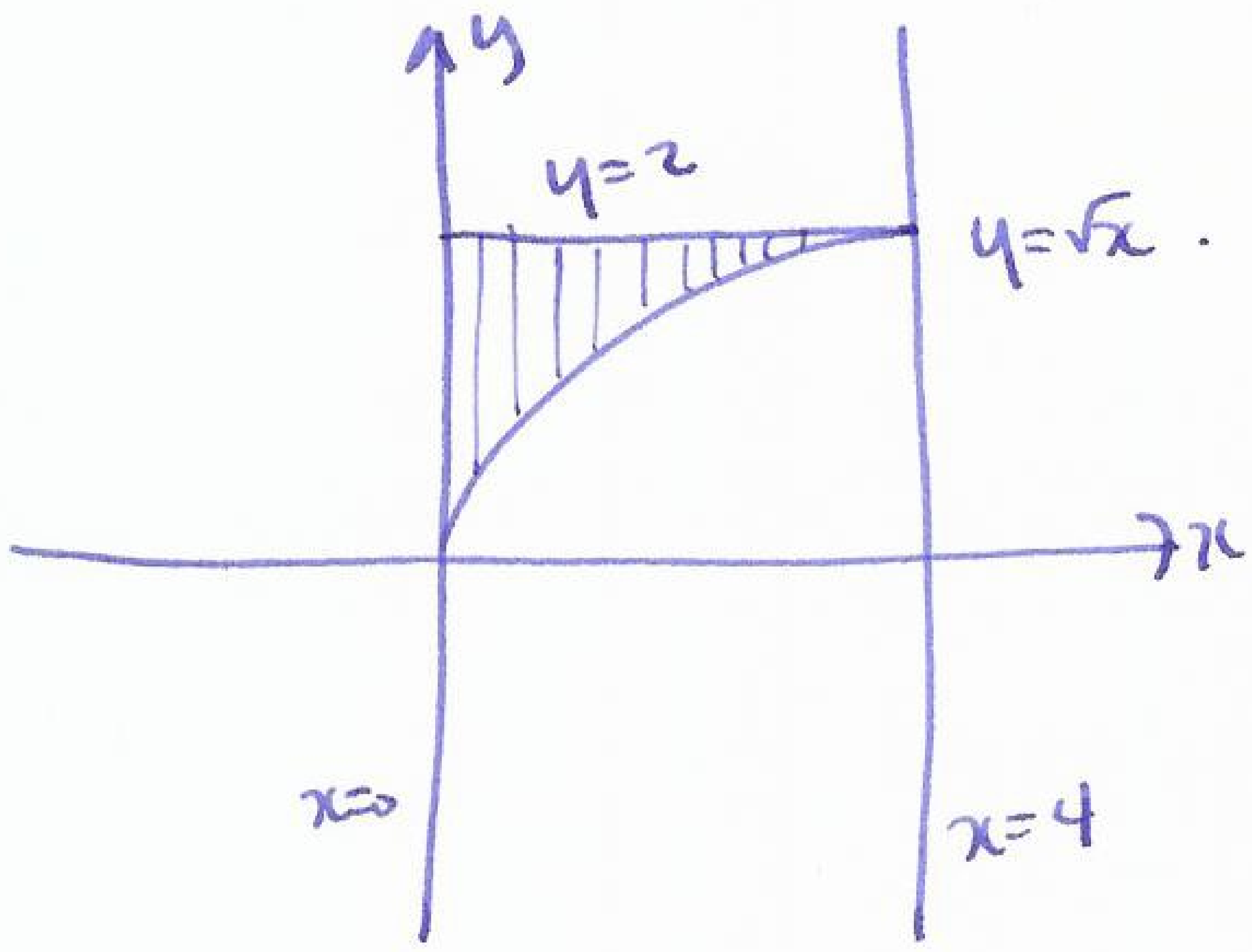
$$= 8 - 4x + \cancel{\frac{1}{2}}x^2 - 4x + x^2 - \cancel{\frac{1}{2}}x^2 + x^2 = 2x^2 - 8x + 8$$

$$\int_0^2 2x^2 - 8x + 8 \; dx = \left[\frac{2}{3}x^3 - 4x^2 + 8x \right]_0^2$$

$$= \frac{16}{3} - 16 + 16 = \frac{16}{3}$$

(10) Evaluate the following integral by changing the order of integration.

$$\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3)dydx$$



$$\int_0^2 \int_0^{y^2} \sin(y^3)\, dx\, dy$$

$$\left[x \sin(y^3)\right]_0^{y^2} = y^2 \sin(y^3).$$

$$\left[-\frac{1}{3}\cos(y^3)\right]_0^2 = -\frac{1}{3}\cos(8) + \frac{1}{3}$$