Math 233 Calculus 3 Spring 13 Midterm 2a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the tangent plane to the surface $z = x^2y - xy + 2$ at the point $(1, -2, 2)$.

$$z = L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$f_x = 2xy - y \qquad f_x(1,-2) = -2$$

$$f_y = x^2 - x \qquad f_y(1,-2) = 0$$

$$z = 2 - 2(x-1) + 0(y+2)$$

$$z = 4 - 2x$$

(2) (10 points) Find the linear approximation to the function $f(x, y, z) = e^{xy} + \tan^{-1}(2y + 3z)$ at the point $(1, 2, -1)$.

$$L(x,y,z) = f(a,b,c) + f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$$

$$f_x = ye^{xy} + 0 \qquad f_x(1,2,-1) = 2e^2$$

$$f_y = xe^{xy} + \frac{1}{1+(2y+3z)^2}\cdot 2 \qquad f_y(1,2,-1) = e^2 + \frac{2}{2}$$

$$f_z = \frac{1}{1+(2y+3z)^2}\cdot 3 \qquad f_z(1,2,-1) = \frac{3}{2}$$

$$L(x,y,z) = e^2 + \tan^{-1}(-1) + 2e^2(x-1) + (e^2+1)(y-2) + \frac{3}{2}(z+1)$$

(3) (10 points) Find the gradient vector for the function $f(x, y, z) = x^2 + y^2 - z$ at the point $(2, 1, 4)$. Use this to find the tangent plane to the surface $z = x^2 + y^2 + 1$ at the point $(2, 1, 4)$.

$\nabla f = \langle 2x, 2y, -1 \rangle$

$\nabla f(2,1,4) = \langle 4, 2, -1 \rangle$

$4x + 2y - z = d$ through $(2,1,4)$

$8 + 2 - 4 = d$

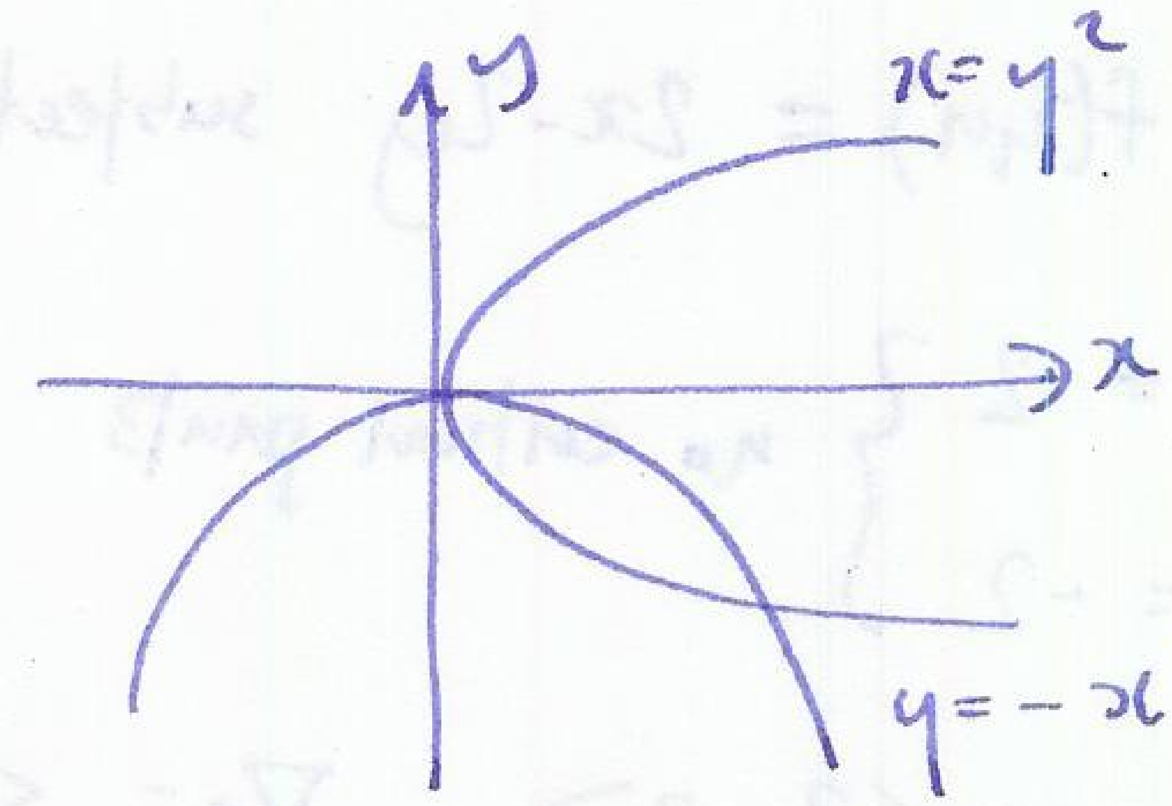
tangent plane: $z = -6 + 4x + 2y$

(4) (10 points) Find the critical points of $f(x, y) = x^3 - y^3 + 3xy$, and use the second derivative test to classify them, if possible.

$f_x = 3x^2 + 3y = 0$: $y = -x^2$

$f_y = -3y^2 + 3x = 0$: $x = y^2$

critical points $(0,0)$, $(1,-1)$



$f_{xx} = 6x$

$f_{xy} = 3$

$f_{yy} = -6y$

$D(0,0) = 0 - 3^2 = -9 < 0$ saddle

$D(1,-1) = 6 \cdot 6 - 3^2 = 36 - 9 = 27 > 0$

$f_{xx} > 0$ minimum

(5) (10 points) Find the maximum and minimum values of $2x - 2y$ on the disc $x^2 + y^2 \leq 4$.

$f(x,y) = 2x - 2y$ subject to $g(x,y) = x^2 + y^2 \leq 4$

$\left.\begin{array}{l} f_x = 2 \\ f_y = -2 \end{array}\right\}$ no critical points

$\nabla f = \langle 2, -2 \rangle \quad \nabla g = \langle 2x, 2y \rangle$

$\nabla f = \lambda \nabla g \quad \left.\begin{array}{l} 2 = \lambda 2x \\ -2 = \lambda 2y \end{array}\right\} \; x = -y \quad x^2 + y^2 = 4 \quad \Big\} \; x^2 + x^2 = 4 \quad x = \pm\sqrt{2}.$

$(\sqrt{2}, -\sqrt{2}) \quad (-\sqrt{2}, +\sqrt{2})$

$f(\sqrt{2}, -\sqrt{2}) = 4\sqrt{2}$ max

$f(-\sqrt{2}, \sqrt{2}) = -4\sqrt{2}$ min

(6) (10 points) Use Lagrange's method to find the closest point to the origin for the plane $x + 2y + 3z = 6$.

min $\underbrace{d^2 = x^2+y^2+z^2}_{f(x,y,z)}$ subject to $\underbrace{x+2y+3z=6}_{g(x,y,z)}$.

$\nabla f = \langle 2x, 2y, 2z \rangle$ $\qquad \nabla g = \langle 1,2,3 \rangle$

$\nabla f = \lambda \nabla g$ $\quad 2x = \lambda$

$2y = 2\lambda$

$2z = 3\lambda$

$x + 2y + 3z = 6$ $\qquad \frac{\lambda}{2} + 2\lambda + \frac{9\lambda}{2} = 6$

$7\lambda = 6 \quad \lambda = 6/7$

$\left(\frac{3}{7}, \frac{6}{7}, \frac{9}{7}\right)$

(7) (10 points) Evaluate

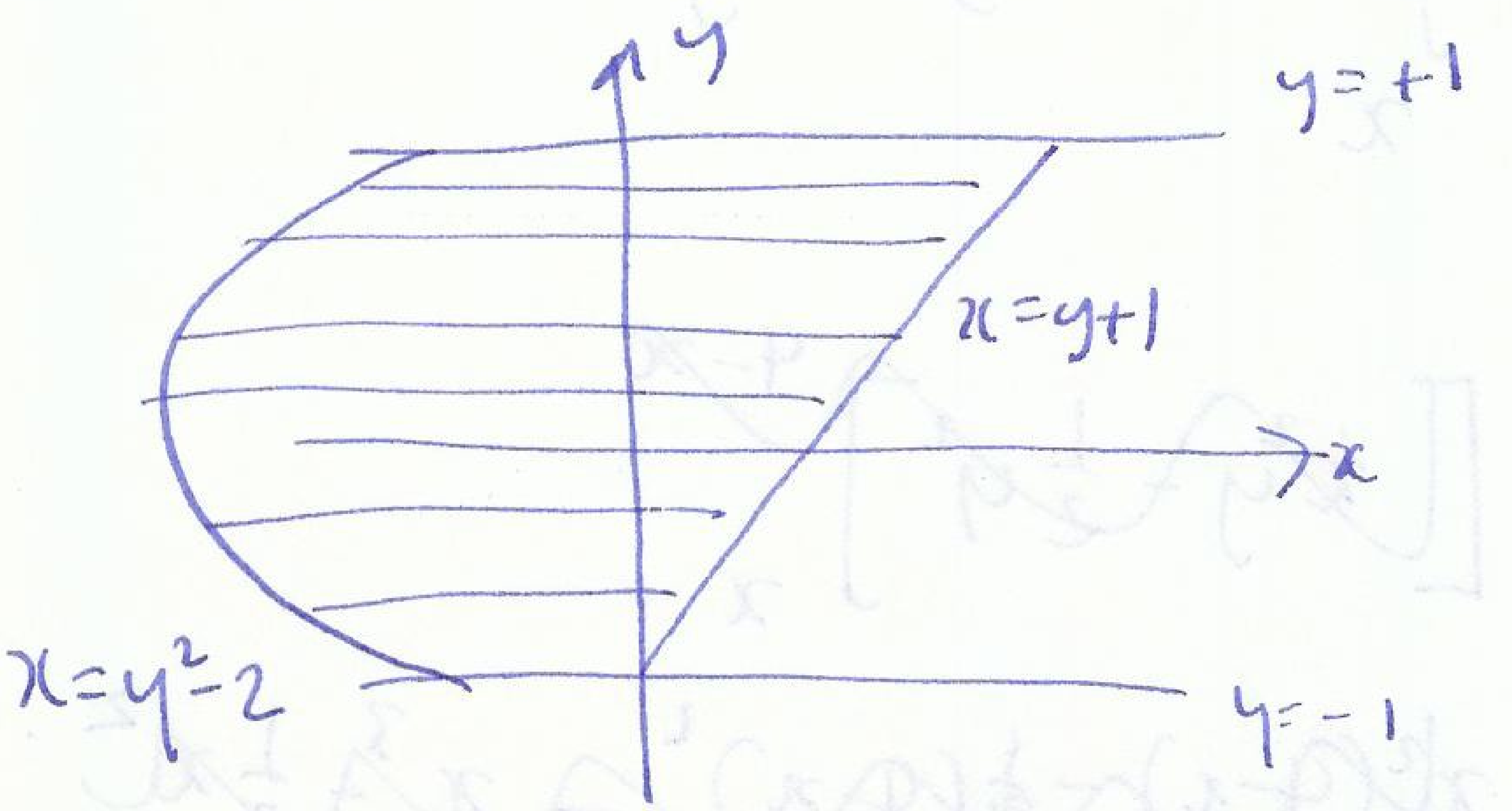
$$\int_0^2 \int_x^3 e^{2x+3y}\,dy\,dx$$

$$\left[\frac{1}{3}e^{2x+3y}\right]_x^3 = \frac{1}{3}e^{2x+9} - \frac{1}{3}e^{5x}$$

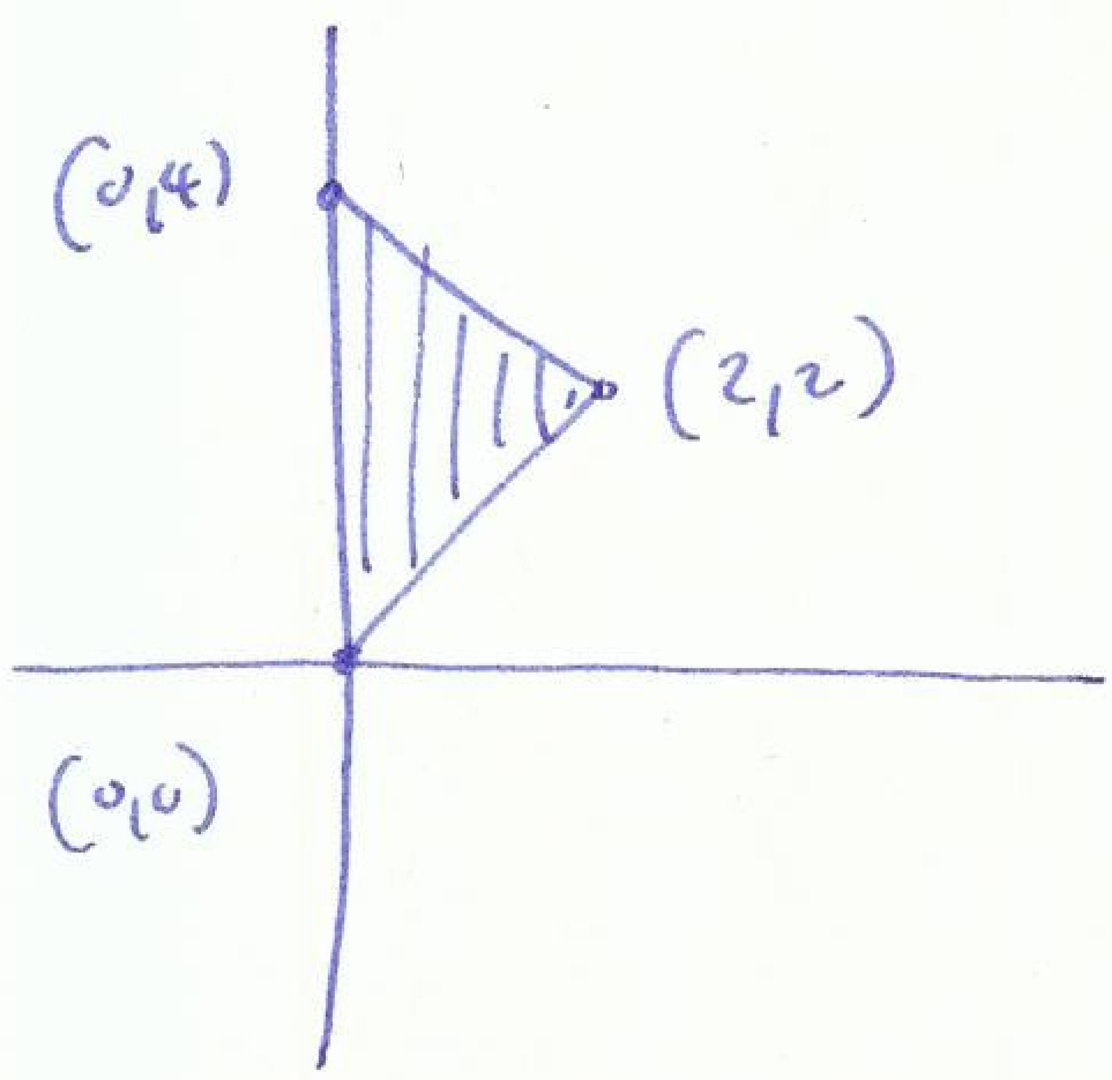
$$\left[\frac{1}{6}e^{2x+9} - \frac{1}{15}e^{5x}\right]_0^2 = \frac{1}{6}e^{13} - \frac{1}{15}e^{10} - \frac{1}{6}e^{9} + \frac{1}{15}$$

(8) Draw the region in the plane corresponding to the limits of the following double integral.

$$\int_{-1}^{1}\int_{y^2-2}^{y+1} f(x,y)\,dx\,dy$$



(9) Find the integral of the function $f(x, y) = x^2 - y$ over the triangle with vertices $(0, 0)$, $(0, 4)$ and $(2, 2)$.



$$\int_0^2 \int_x^{4-x} x^1 - y \; dy\, dx$$

$$\cancel{\left[x^2 y - \tfrac{1}{2} y^2\right]_x^{4-x}}$$

$$\left[xy - \tfrac{1}{2}y^2\right]_x^{4-x}.$$

$$\cancel{x^2(4-x) - \tfrac{1}{4}(4-x)^2 - x^3 + \tfrac{1}{2}x^2}.$$

$$\cancel{x^2(4-x) - \tfrac{1}{2}(4-x)^2 - x^3 - \tfrac{1}{2}x^2}.$$

$$x(4-x) - \tfrac{1}{2}(4-x)^2 - x^2 + \tfrac{1}{2}x^2$$

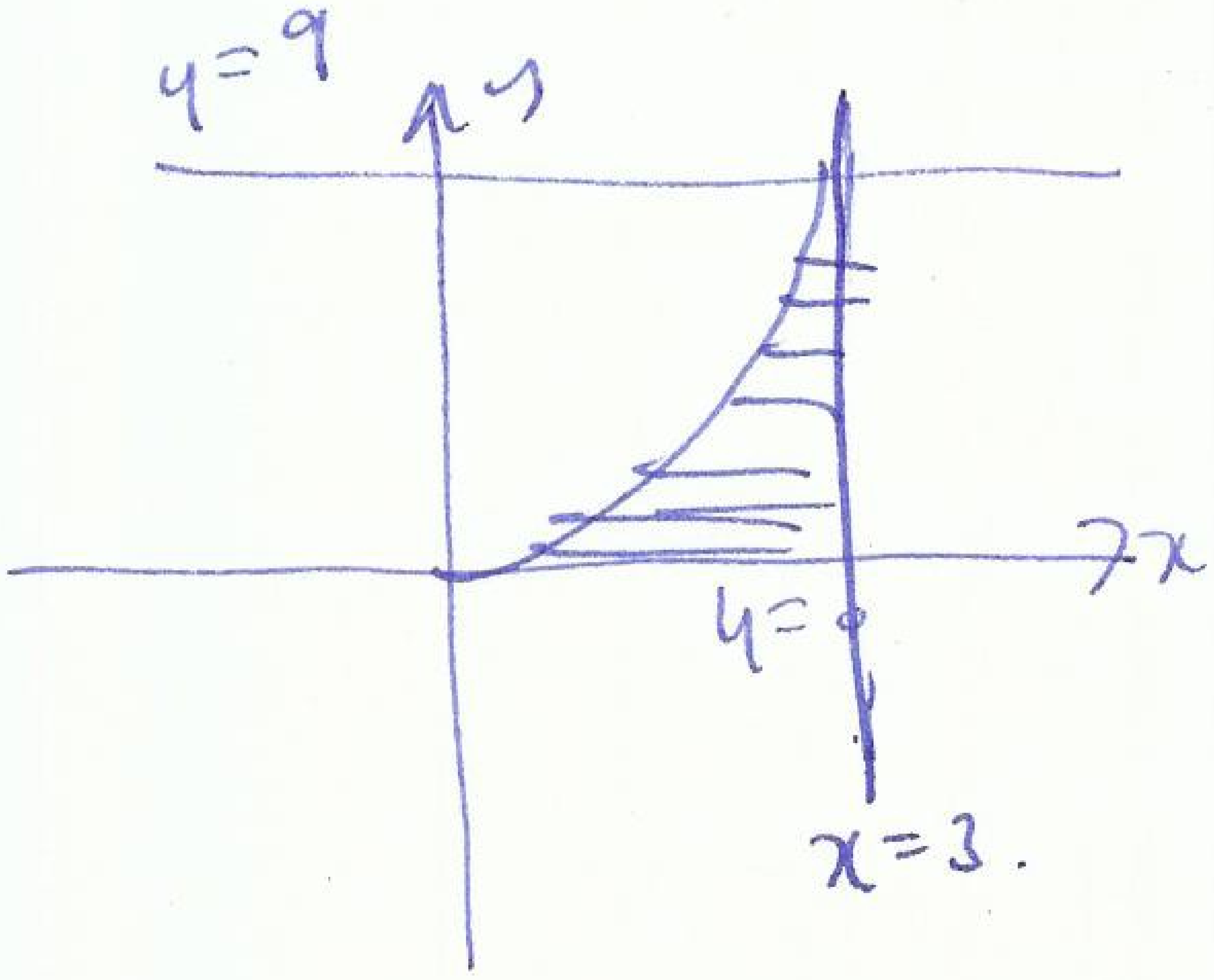
$$4x - x^2 - 8 + 4x - \cancel{\tfrac{1}{2}}x^2 - x^2 + \cancel{\tfrac{1}{2}}x^2$$

$$\int_0^2 -2x^2 + 8x - 8 \; dx = \left[-\tfrac{2}{3}x^3 + 4x^2 - 8x\right]_0^2.$$

$$-\frac{16}{3} + \cancel{16} - \cancel{16} = -\frac{16}{3}.$$

(10) Evaluate the following integral by changing the order of integration.

$$\int_0^9 \int_{\sqrt{y}}^3 \sin(x^3)\,dx\,dy$$



$x=\sqrt{y}$

$x^2=y$

$$\int_0^3 \int_0^{x^2} \sin(x^3)\,dy\,dx.$$

$$\left[y\sin(x^3)\right]_0^{x^2} = x^2\sin(x^3).$$

$$\left[-\frac{1}{3}\cos(x^3)\right]_0^3.$$

$$= -\frac{1}{3}\cos(9) + \frac{1}{3}$$