Math 233 Calculus 3 Spring 13 Sample Midterm 2

(1) Find the equation of the tangent plane to the surface $z = x^2 - 4y^2$ at the point $(2, 1, -1)$.

(2) Find the linear approximation to the function $f(x, y, z) = e^{3xz} + \ln(2y + z)$ at the point $(1, 2, 4)$.

(3) You are standing on a surface given by the equation $z = 4x^2 - y^2$. If you're standing at the point $(1, 2, 0)$, in which direction is the fastest way up?

(4) The temperature in the solar system is given by

$$T(x, y, z) = \frac{10^5}{x^2 + y^2 + z^2}$$

If a comet travels along the path $\mathbf{r}(t) = (2t, t^2 - 8, t)$, how fast is the temperature changing when $t = 2$?

(5) Find the critical points of the following functions, and use the second derivative test to classify them, if possible.

(a)

$$f(x, y) = x^3 - 3xy + y^3$$

(b)

$$f(x, y) = e^x - 2xe^y$$

(c)

$$f(x, y) = x \ln(x + y)$$

(6) Find the extreme values of $f(x, y) = 4x^2 - 2y^2$ on the square $0 \leq x \leq 1, 0 \leq y \leq 1$.

(7) Use Lagrange multipliers to find the minimum and maximum values of $x^2y + x + y$ subject to $xy = 4$.

(8) Use Lagrange multipliers to find the dimensions of the cylindrical tin can of volume V with least surface area.

(9) Integrate the function $f(x, y) = xy$ over the triangle in the xy-plane with vertices $(0, 4), (2, 0)$ and $(2, 4)$.

(10) Evaluate the following integral by changing the order of integration.:

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{y}{(1+x^2+y^2)^2} dxdy$$

(11) Write down limits for an integral over the tetrahedron with vertices $(0,0,0)$, $(0,1,0)$, $(1,0,1)$ and $(1,1,1)$.