Math 233 Calculus 3 Spring 13 Midterm 1b

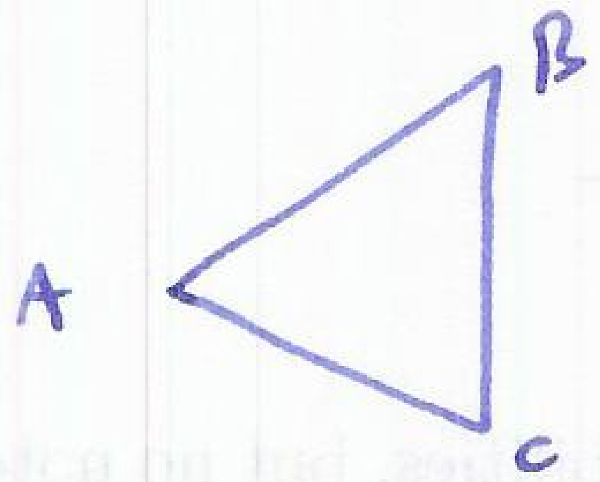
Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find the area of the triangle with vertices $(4, 1, -2)$, $(3, 2, 0)$ and $(-2, 3, 5)$.



$$\text{area} = \frac{1}{2} \| \vec{AB} \times \vec{AC} \|$$

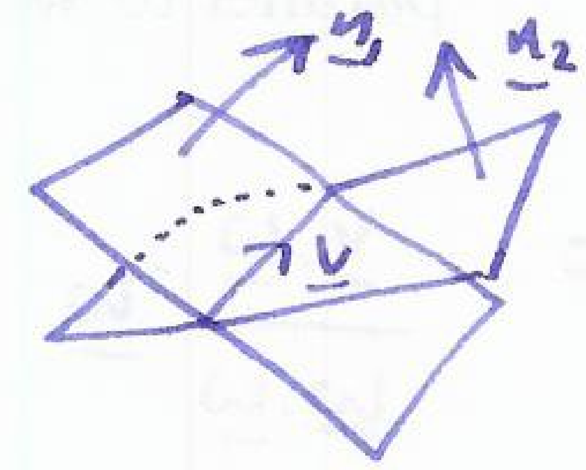
$$\vec{AB} = \langle -1, 1, 2 \rangle$$

$$\vec{AC} = \langle -6, +2, 7 \rangle$$

$$\begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ -6 & +2 & 7 \end{vmatrix} = i \begin{vmatrix} 1 & 2 \\ +2 & 7 \end{vmatrix} - j \begin{vmatrix} -1 & 2 \\ -6 & 7 \end{vmatrix} + k \begin{vmatrix} -1 & 1 \\ -6 & +2 \end{vmatrix} = \langle \overset{3}{\cancel{11}}, -5, \overset{4}{\cancel{8}} \rangle$$

$$\text{area} = \frac{1}{2} \sqrt{\underset{3^2}{\cancel{11^2}} + 5^2 + \underset{4^2}{\cancel{8^2}}}$$

(2) (10 points) Find a parametric equation for the line of intersection of the two planes $-2x + y - z = 4$ and $x - y + 4z = 2$.



$\underline{n}_1 = \langle -2, 1, -1 \rangle$

$\underline{n}_2 = \langle 1, -1, 4 \rangle$

$$\underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -2 & 1 & -1 \\ 1 & -1 & 4 \end{vmatrix} = \underline{i} \begin{vmatrix} 1 & -1 \\ -1 & 4 \end{vmatrix} - \underline{j} \begin{vmatrix} -2 & -1 \\ 1 & 4 \end{vmatrix} + \underline{k} \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= \langle 3, 5, 1 \rangle = \underline{v}$$

point on line: choose $z = 0$: $\left.\begin{aligned} -2x + y &= 4 \\ x - y &= 2 \end{aligned}\right\}$ $\quad -y = 8$, $\quad x = -6$

$\underline{r}(t) = \langle -6, -8, 0 \rangle + t \langle 3, 5, 1 \rangle$

(3) (10 points) Write the vector $\mathbf{v} = \langle 1, 5, -2 \rangle$ as a sum of two vectors, one parallel to $\mathbf{w} = \langle 3, 1, -1 \rangle$, and one perpendicular to $\mathbf{w}$.

$$\text{proj}_{\underline{w}} \underline{v} = \frac{\underline{v} \cdot \underline{w}}{\underline{w} \cdot \underline{w}} \underline{w} = \frac{3+5+2}{9+1+1} \langle 3, 1, -1 \rangle = \frac{10}{11} \langle 3, 1, -1 \rangle$$

parallel vector

$$\text{perpendicular vector} = \underline{v} - \text{proj}_w \underline{v}$$

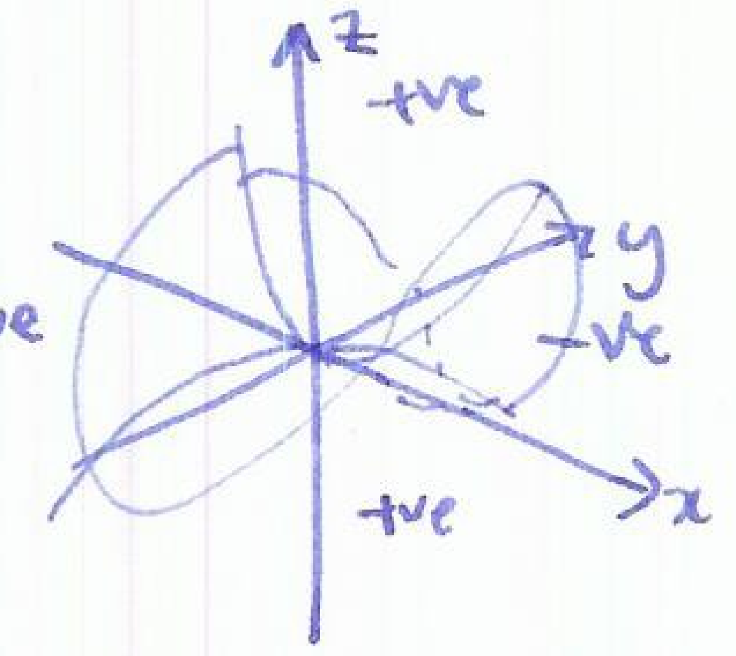
$$= \langle 1, 5, -2 \rangle - \frac{10}{11} \langle 3, 1, -1 \rangle = \left\langle \frac{-19}{11}, \frac{45}{11}, \frac{-12}{11} \right\rangle$$

(4) (10 points) Sketch the graphs of the following functions, and their contour maps/level sets.

(a) $f(x, y) = -xy$

(b) $f(x, y) = x^2 + y^2 - 1$

a)

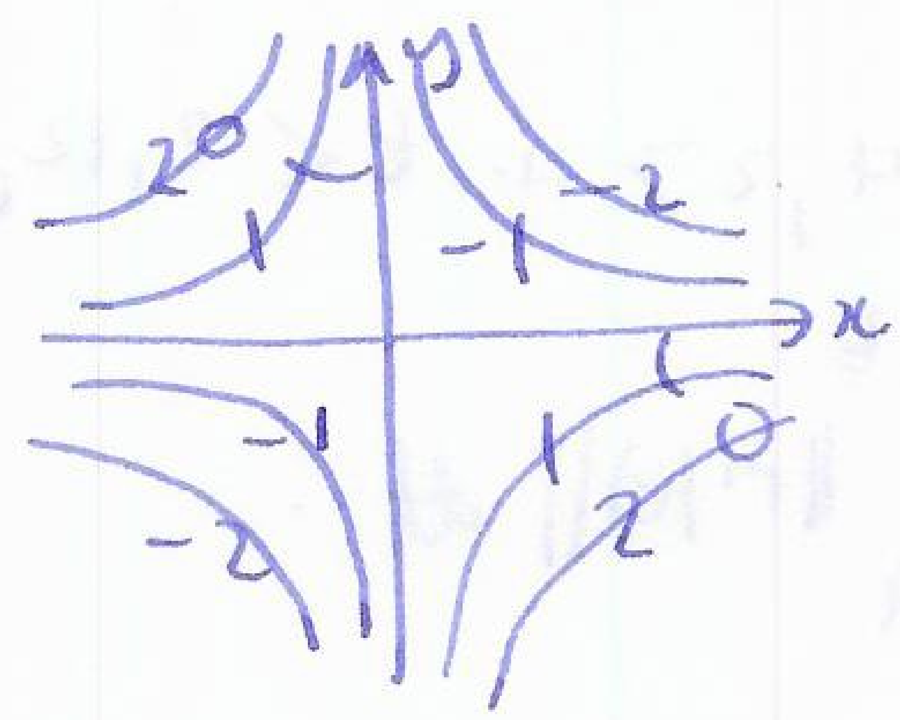


graph

level sets.

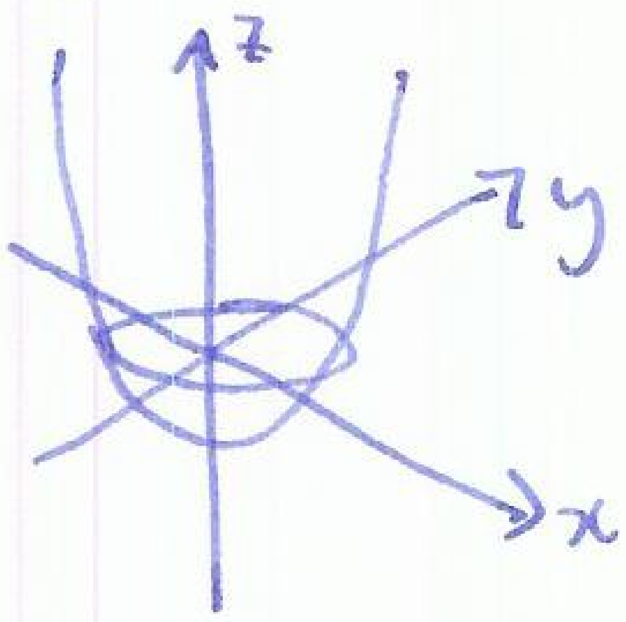
$-xy = c$

$y = -\frac{c}{x}$.



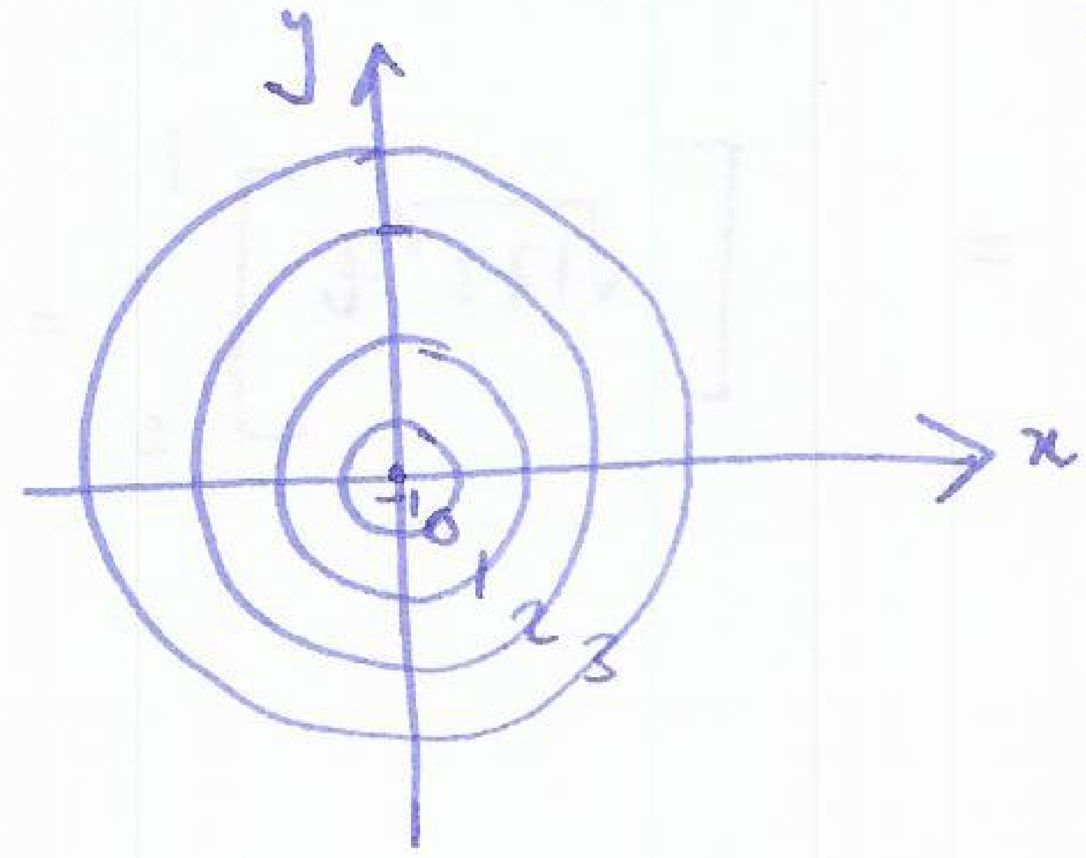
b)

graph



level sets

$x^2 + y^2 - 1 = c$



(5) (10 points) Write down a parameterization for the straight line segment from $(3, -4, 2)$ to $(5, 8, -3)$. Use the integral formula for arc length to find the length of this line.

$\underline{v} = \langle 2, 12, -5 \rangle$

$\underline{r}(t) = \langle 3, -4, 2 \rangle + t\langle 2, 12, -5 \rangle$

arc length $= \int_a^b \|r'(t)\| \, dt$.

$\underline{r}'(t) = \langle 2, 12, -5 \rangle \qquad \|r'(t)\| = \sqrt{4+144+25}$.

arc length $= \int_0^1 \sqrt{173} \, dt = \left[\sqrt{173}\, t \right]_0^1 = \sqrt{173}$.

(6) (10 points) The position of a particle is given by $\mathbf{r}(t) = \langle \ln(t+2), e^{-2t}, \tan(t/3) \rangle$, find the acceleration of the particle.

$$\underline{r}'(t) = \left< \frac{1}{t+2},\ -2e^{-2t},\ \sec^2\left(\frac{t}{3}\right) \cdot \frac{1}{3} \right>$$

$$\underline{r}''(t) = \left< -(t+2)^{-2},\ 4e^{-2t},\ \frac{1}{3} \cdot 2\sec\left(\frac{t}{3}\right) \cdot \sec\left(\frac{t}{3}\right)\tan\left(\frac{t}{3}\right) \cdot \frac{1}{3} \right>$$

(7) (10 points) An object is thrown from the origin with initial velocity $\langle 20, 5, 10\rangle$ m/s. Find an expression for the position of the object at time t it moves under the gravitational force $\mathbf{F} = \langle 0, 0, -gm\rangle$ m/s^2. Feel free to take $g = 10$.

$\underline{F} = m\underline{a} = m\underline{r}''(t)$

$\langle 0, 0, -10m\rangle$.

$\underline{r}''(t) = \langle 0, 0, -10\rangle$

$\underline{r}'(t) = \langle 0, 0, -10t\rangle + \underline{c}$

$\underline{r}'(0) = \langle 20, 5, 10\rangle = \underline{c}$

$\underline{r}'(t) = \langle 0, 0, -10t\rangle + \langle 20, 5, 10\rangle$

$\underline{r}(t) = \langle 0, 0, -5t^2\rangle + \langle 20, 5, 10\rangle t + \underline{d}$

$\underline{r}(0) = \langle 0, 0, 0\rangle = \underline{d}$

$\underline{r}(t) = \langle 0, 0, -5t^2\rangle + t\langle 20, 5, 10\rangle + \langle 0, 0, 0\rangle$.

(8) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y+xy}{x^2+y^2}$$

$$\lim_{(x,0)\to(0,0)} \frac{x^2y+xy}{x^2+y^2} = \lim_{x\to 0} \frac{0}{x^2} = 0$$

try $x=y$: $$\lim_{(x,x)\to(0,0)} \frac{x^2y+xy}{x^2+y^2} = \lim_{x\to 0} \frac{x^3+x^2}{2x^2} = \lim_{x\to 0} \frac{1+x}{2} = \frac{1}{2} \neq 0$$

DNE.

(9) Find all first partial derivatives for $f(x, y, z) = \sqrt{5z^2 - 2xy}$.

$$\frac{\partial f}{\partial x} = \frac{1}{2}\left(5z^2 - 2xy\right)^{-1/2} \cdot (-2y)$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}\left(5z^2 - 2xy\right)^{-1/2} \cdot (-2x)$$

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(5z^2 - 2xy\right)^{-1/2} \cdot 10z \,.$$

(10) Find f_{xz} for $f(x,y,z) = \tan^{-1}(3xy - 2xz)$.

$$\frac{\partial f}{\partial x} = \frac{1}{1+(3xy-2xz)^2} \cdot (3y-2z)$$

$$\frac{\partial^2 f}{\partial z \partial x} = \frac{\left(1+(3xy-2xz)^2\right)(-2) - (3y-2z)\left(2(3xy-2xz)\cdot(-2x)\right)}{\left(1+(3xy-2xz)^2\right)^2}.$$