**Math 233 Calculus 3 Spring 13 Midterm 1a**

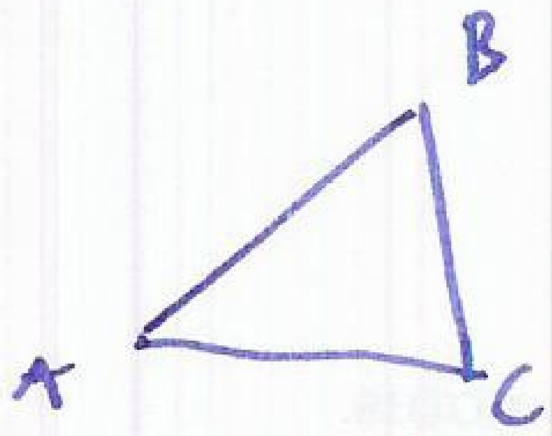
Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator without symbolic algebra capabilities, but no notes.

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| | 80 | |

| | |
|---|---|
| Midterm 1 | |
| Overall | |

(1) (10 points) Find the area of the triangle with vertices $(1, 4, -1)$, $(1, 0, 2)$ and $(-2, -3, -3)$.




$$\text{area} = \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \| \qquad AB = \langle 0, -4, 3 \rangle$$

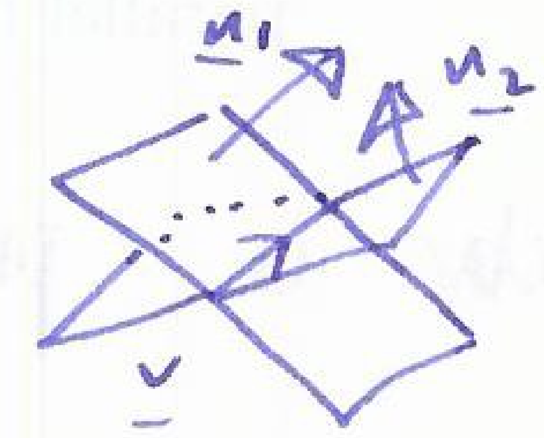
$$AC = \langle -3, -7, -2 \rangle$$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -4 & 3 \\ -3 & -7 & -2 \end{vmatrix} = \underline{i} \begin{vmatrix} -4 & 3 \\ -7 & -2 \end{vmatrix} - \underline{j} \begin{vmatrix} 0 & 3 \\ -3 & -2 \end{vmatrix} + \underline{k} \begin{vmatrix} 0 & -4 \\ -3 & -7 \end{vmatrix}$$

$$= \langle 29, -9, -12 \rangle$$

$$\text{area} = \frac{1}{2} \sqrt{29^2 + 9^2 + 12^2}$$

(2) (10 points) Find a parametric equation for the line of intersection of the two planes $x - 2y - z = 1$ and $x + 2y + 2z = 4$.




$\underline{n}_1 = \langle 1, -2, -1 \rangle$

$\underline{n}_2 = \langle 1, 2, 2 \rangle$ $\quad\quad \underline{v} = \underline{n}_1 \times \underline{n}_2$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -2 & -1 \\ 1 & 2 & 2 \end{vmatrix} = \underline{i} \begin{vmatrix} -2 & -1 \\ 2 & 2 \end{vmatrix} - \underline{j} \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} + \underline{k} \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix}$$

$$= \langle -2, -3, 4 \rangle$$

find point on line: try $z = 0$

$$\left.\begin{aligned} x - 2y &= 1 \\ x + 2y &= 4 \end{aligned}\right\} \quad 2x = 5 \quad x = 5/2, \quad y = 3/4$$

$$\underline{r}(t) = \langle \tfrac{5}{2}, \tfrac{3}{4}, 0 \rangle + t \langle -2, -3, 4 \rangle$$

(3) (10 points) Write the vector $\mathbf{v} = \langle -3, 2, -1 \rangle$ as a sum of two vectors, one parallel to $\mathbf{w} = \langle 2, 3, 1 \rangle$, and one perpendicular to $\mathbf{w}$.

parallel vector is $\text{proj}_{\underline{w}}(\underline{v}) = \dfrac{\underline{v}\cdot\underline{w}}{\underline{w}\cdot\underline{w}}\,\underline{w} = \dfrac{-6+6-1}{4+9+1}\langle 2,3,1\rangle$

$= -\dfrac{1}{14}\langle 2,3,1\rangle$

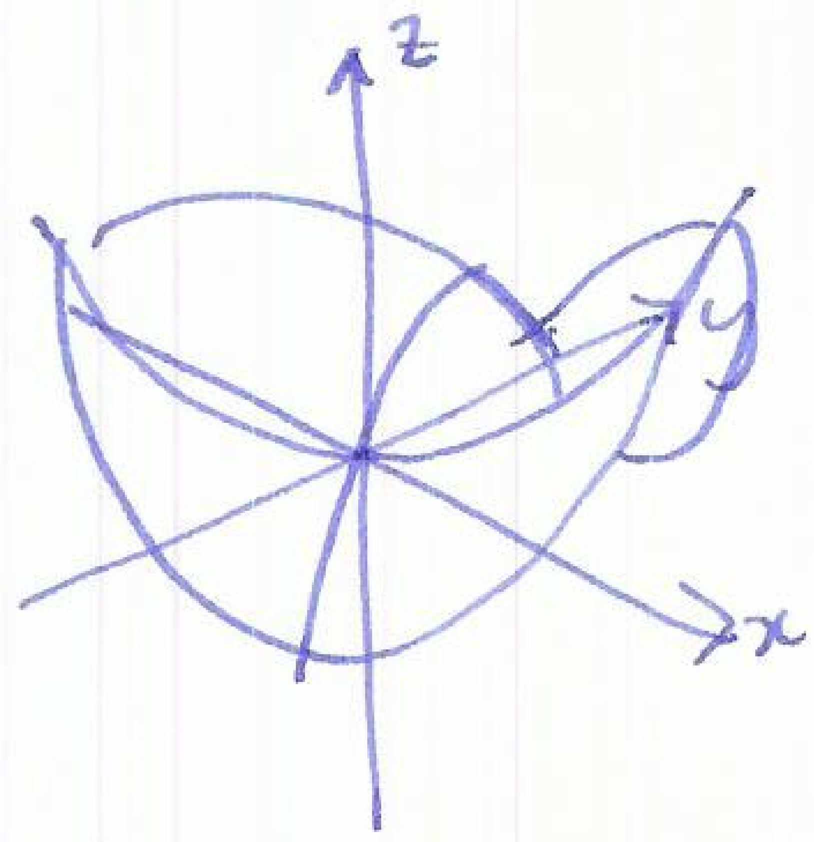
perpendicular vector is $\underline{v} - \text{proj}_{\underline{w}}(\underline{v}) = \langle -3,2,-1\rangle + \dfrac{1}{14}\langle 2,3,1\rangle$

(4) (10 points) Sketch the graphs of the following functions, and their contour maps/level sets.

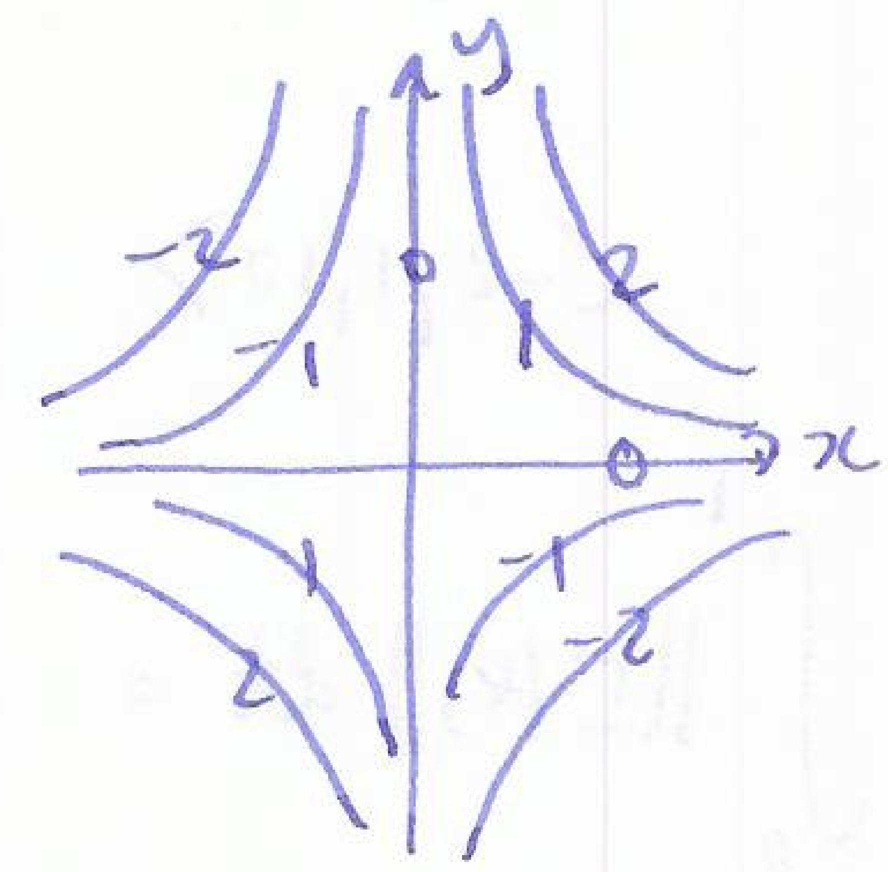
(a) $f(x, y) = xy$

(b) $f(x, y) = 4 - x^2 - y^2$

a) graph



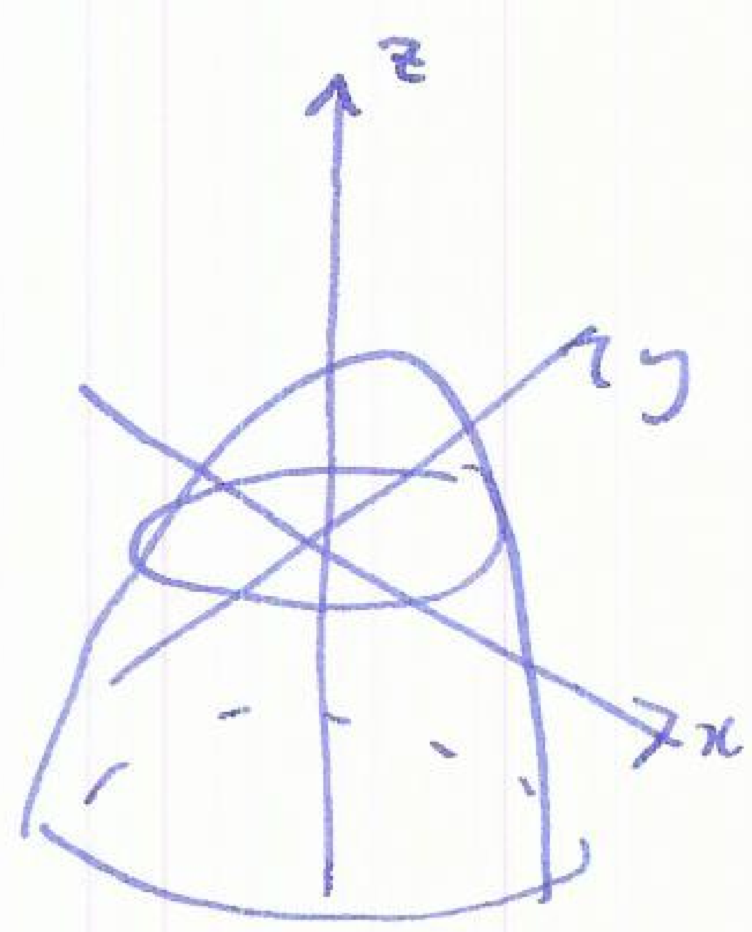

level sets.

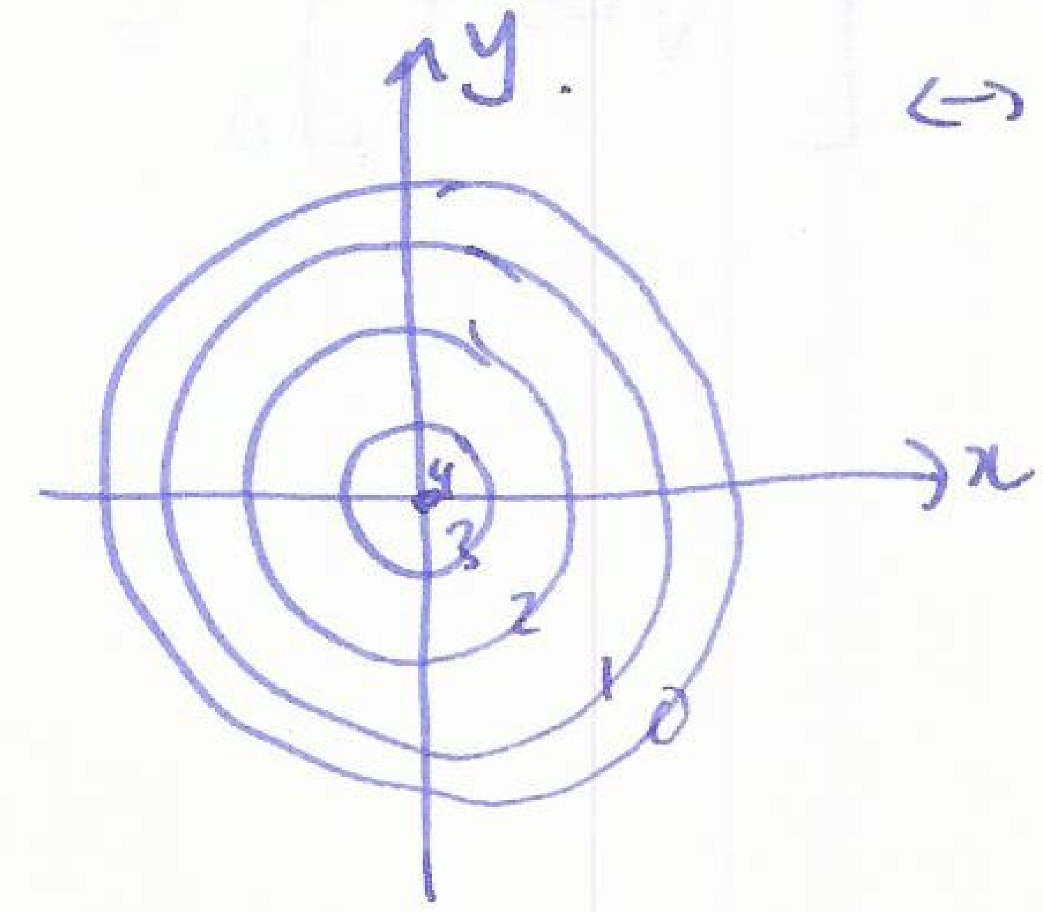



$xy = c$

$\Leftrightarrow y = \frac{c}{x}$

b)




$4 - x^2 - y^2 = c$

$\Leftrightarrow x^2 + y^2 = 4 - c$

(5) (10 points) Write down a parameterization for the straight line segment from $(5, 4, -2)$ to $(3, 5, 8)$. Use the integral formula for arc length to find the length of this line.

$$\underline{r}(t) = \langle 5, 4, -2 \rangle + t\langle -2, 1, 10 \rangle$$

$$\underline{r}'(t) = \langle -2, 1, 10 \rangle$$

$$\text{arc length} = \int_0^1 \|\underline{r}'(t)\|\, dt = \int_0^1 \sqrt{4+1+100}\, dt$$

$$= \Big[\sqrt{105}\, t\Big]_0^1 = \sqrt{105}.$$

(6) (10 points) The position of a particle is given by $\mathbf{r}(t) = \langle \ln(2t+1), e^{-3t}, \tan(t/2) \rangle$, find the acceleration of the particle.

$$\underline{r}'(t) = \left\langle \frac{1}{2t+1} \cdot \cancel{2}\,2,\ -3e^{-3t},\ \sec^2\left(\frac{t}{2}\right) \cdot \frac{1}{2} \right\rangle$$

$$\underline{r}''(t) = \left\langle 2\cancel{2} \cdot -(2t+1)^{-2} \cdot 2,\ 9e^{-3t},\ 2\sec\left(\frac{t}{2}\right) \cdot \sec\left(\frac{t}{2}\right)\tan\left(\frac{t}{2}\right) \cdot \frac{1}{4} \right\rangle$$

(7) (10 points) An object is thrown from the origin with initial velocity $\langle 10, 5, 20\rangle$ m/s. Find an expression for the position of the object at time $t$ it moves under the gravitational force $\mathbf{F} = \langle 0, 0, -gm\rangle$ m/s$^2$. Feel free to take $g = 10$.

$\underline{F} = m\underline{a} = m\,\underline{r}''(t) = \langle 0, 0, -10\rangle$

$\underline{r}'(t) = \langle 0, 0, -10t\rangle + \underline{c}$

$\underline{r}'(0) = \langle 10, 5, 20\rangle = \underline{c}$

$\underline{r}(t) = \langle 0, 0, -\frac{1}{5}t^2\rangle + \langle 10, 5, 20\rangle t + \underline{d}$

$\underline{r}(0) = \langle 0, 0, 0\rangle = \underline{d}$

$\underline{r}(t) = \langle 0, 0, -\frac{1}{5}t^2\rangle + \langle 10, 5, 20\rangle t + \langle 0, 0, 0\rangle.$

(8) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy + x^2y}{x^2 + y^2}$$

$$\left.\begin{aligned} \lim_{(x,0)\to(0,0)} \frac{xy + x^2y}{x^2+y^2} &= \lim_{x\to 0} \frac{0}{x^2} = 0 \\ \lim_{(x,x)\to(0,0)} \frac{xy + x^2y}{x^2+y^2} &= \lim_{x\to 0} \frac{x^2+x^3}{2x^2} = \frac{1}{2} \end{aligned}\right\} \Rightarrow \text{DNE}$$

(9) Find all first partial derivatives for $f(x, y, z) = \sqrt{3xy - 7z^2}$.

$$\frac{\partial f}{\partial x} = \frac{1}{2}\left(3xy - 7z^2\right)^{-1/2} \cdot 3y$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}\left(3xy - 7z^2\right)^{-1/2} \cdot 3x$$

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(3xy - 7z^2\right)^{-1/2} \cdot (-14z)$$

(10) Find $f_{xz}$ for $f(x, y, z) = \tan^{-1}(2xy + 3xz)$.

$$\frac{\partial f}{\partial x} = \frac{1}{1 + (2xy+3xz)^2} \cdot (2y+3z)$$

$$\frac{\partial^2 f}{\partial z \partial x} = \frac{\left(1+(2xy+3xz)^2\right) . 3 - \left(2(2xy+3xz).(3x)\right)(2y+3z)}{\left(1+(2xy+3xz)^2\right)^2} .$$