Sample midterm 1 solutions

Q1 a) $\text{proj}_{\underline{v}}(\underline{u}) = \dfrac{\underline{u}\cdot\underline{v}}{\underline{v}\cdot\underline{v}}\underline{v} = \dfrac{-9-4+4}{9+4+1}\langle -3,2,1\rangle = \dfrac{-9}{14}\langle -3,2,1\rangle$

$\|\text{proj}_{\underline{v}}(\underline{u})\| = \dfrac{9}{14}\sqrt{9+4+1} = \dfrac{9\sqrt{14}}{14}$.

b) $\underline{u}_{\parallel} = -\dfrac{9}{14}\langle -3,2,1\rangle \qquad \underline{u}_{\perp} = \langle 3,-2,4\rangle + \dfrac{9}{14}\langle -3,2,1\rangle$

Q2 a) area of triangle $= \frac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\|$ $\quad \overrightarrow{AB} = \langle 2,-1,4\rangle$, $\overrightarrow{AC} = \langle 3,3,7\rangle$.

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -1 & 4 \\ 3 & 3 & 7 \end{vmatrix} = \begin{vmatrix} -1 & 4 \\ 3 & 7 \end{vmatrix}\underline{i} - \underline{j}\begin{vmatrix} 2 & 4 \\ 3 & 7 \end{vmatrix} + \underline{k}\begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} = \langle -19,-2,3\rangle.$$

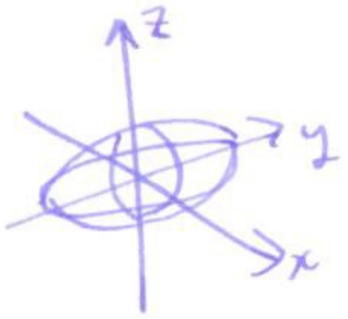
area $= \frac{1}{2}\sqrt{19^2+4^2+3^2}$.

b) $\underline{n} = \langle -19,-2,3\rangle$ contains $(1,-1,2)$.

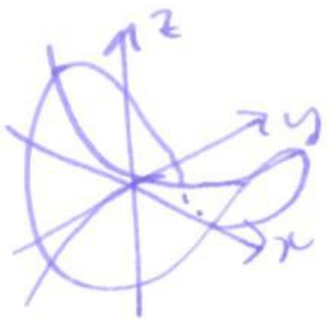
$-19x-2y+3z = -11$

Q3 $\underline{n} = \langle 3,1,-2\rangle$ contains $(3,-1,2)$. $\quad 3x+y-2z = 4$

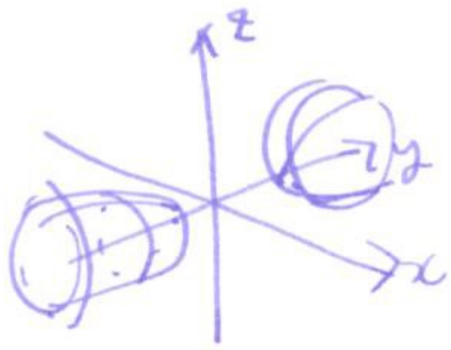
Q4 a)



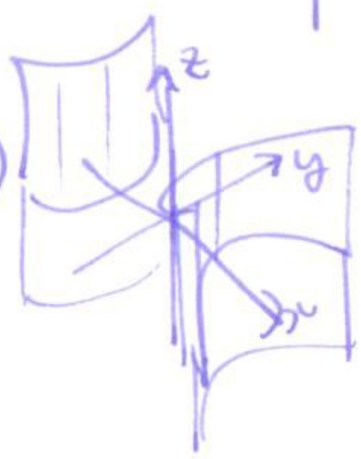
b)



c)



d)



$4x^2 = 72+4y^2$.

Q5 $\underline{a}(t) = \underline{r}''(t) = \langle 6t, 12t^2, -3\rangle$.

$\underline{r}'(t) = \langle 3t^2, 4t^3, -3t\rangle + \underline{c} \qquad \underline{r}'(0) = \underline{c} = \langle 4,-3,-3\rangle$

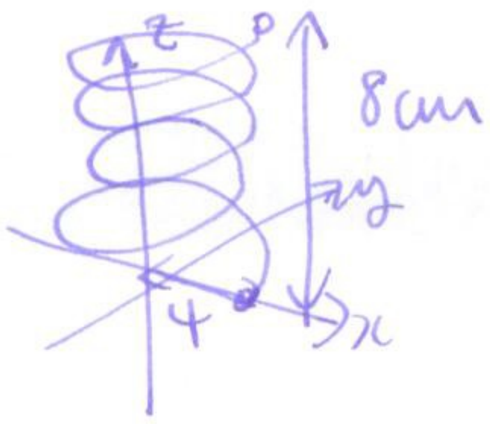
$\underline{r}(t) = \langle \frac{3}{2}t^3, t^4, -\frac{3}{2}t^2\rangle + t\langle 4,-3,-3\rangle + \underline{d}$

$\underline{r}(0) = \underline{d} = \langle 2,2,-3\rangle$.

$\underline{r}(t) = \langle t^3, t^4, -\frac{3}{2}t^2\rangle + t\langle 4,-3,-3\rangle + \langle 2,2,-3\rangle$

$\underline{r}(2) = \langle 8,16,-6\rangle + \langle 8,-6,-6\rangle + \langle 2,2,-3\rangle = \langle 18,12,-15\rangle$

Q6



a) $\underline{r}(t) = \langle 4\cos t, 4\sin t, \frac{t}{\pi} \rangle$

$0 \le t \le 8\pi$

b) $\underline{r}'(t) = \langle -4\sin t, 4\cos t, \frac{1}{\pi} \rangle$.

$$||\underline{r}'(t)|| = \sqrt{16\sin^2 t + 16\cos^2 t + \frac{1}{\pi^2}} = \sqrt{16 + \frac{1}{\pi^2}}.$$

$$\int_0^{8\pi} \sqrt{16 + \frac{1}{\pi^2}}\, dt = 8\pi\sqrt{16 + \frac{1}{\pi^2}}.$$

Q7 $\lim_{(x,y)\to(0,0)} \frac{0}{x^2} = 0$ $\lim_{(0,y)\to(0,0)} \frac{-y^2}{y^2} = -1$ $\not\exists$ DNE.

Q8 $f(x,y,z) = ye^{x+yz} + (x+z)\cos(y+z)$

$f_x = ye^{x+yz} + \cos(y+z)$

$f_y = e^{x+yz} + ye^{x+yz}\cdot z + (x+z)(-\sin(y+z))$

$f_z = y^2 e^{x+yz} + \cos(y+z) + (x+z)(-\sin(y+z))$.

$f_{xx} = ye^{x+yz}$

$f_{yy} = ze^{x+yz} + ze^{x+yz} + ye^{x+yz}\cdot z^2 + (x+z)(-\cos(y+z))$.

$f_{zz} = y^3 e^{x+yz} + -\sin(y+z) + (-\sin(y+z)) + (x+z)(-\cos(y+z))$.

$f_{xy} = e^{x+yz} + yze^{x+yz} + -\sin(y+z)$

$f_{yz} = ye^{x+yz} + y^2 e^{x+yz} + zy^2 e^{x+yz} + (-\sin(y+z)) + (x+z)(-\cos(y+z))$.

$f_{xz} = y^2 e^{x+yz} - \sin(y+z)$.

Q9