Math 233 Calculus 3 Spring 13 Sample Final

(1) Find the line of intersection between the planes $z = 2x - 3y + 2$ and $2x - y - z = 4$.

(2) Sketch the level sets of the function $f(x, y, z) = x^2 + y^2 - z^2$, and calculate the gradient vector at the point $(2, 2, 2)$. Use this to find the tangent plane to $x^2 + y^2 = z^2 + 4$.

(3) You are driving anticlockwise around a circular roundabout of radius 20m, at 5m/s. When your car is facing due north, you throw a tennis ball from the car due east at 10m/s, at an angle of $\pi/3$ from horizontal. Where does the tennis ball land?

(4) Let $f(x, y) = 2x^2 + y^2 - 4y + 3$.
   (a) Find the critical points of f in the region $x^2 + y^2 < 9$, and use the scond derivative test to classify them.
   (b) Use Lagrange multipliers to find the extreme points on the boundary $x^2 + y^2 = 9$.
   (c) Use your answers above to find the extreme values of f on $x^2 + y^2 \leq 9$.

(5) Change the order of integration to evaluate $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 \sin(x^4)\, dx\, dy$.

(6) Write down triple integrals over the following regions.
   (a) The spherical wedge inside the sphere $x^2 + y^2 + x^2 = 9$ cut out by the planes $z = 0, z = 2$, $x = y$ and $x = 0$, containing the point $(1, 2, 0)$.
   (b) The volume inside the cylinder $x^2 + y^2 \leq 4$, above $z = 0$ and below $x + 2y + 4z = 12$.
   (c) The volume of $z = 8 - x^2 - 2y^2$ in the positive octant.

(7) Evaluate $\displaystyle\int\int_R (x - y)\sin(x + y)\, dA$, where R is the square with vertices $(\pi, 0), (2\pi, \pi), (\pi, 2\pi)$ and $(0, \pi)$, using the map $T(u, v) = ((u + v)/2, (u - v)/2)$.

(8) Let C be the boundary of the triangle in the plane with vertices $(0, 0)$, $(1, 0)$ and $(1, 3)$. If $\mathbf{F} = \langle \sqrt{1 + x^3}, 2xy \rangle$, use Green's Theorem to evaluate

$$\int_C \mathbf{F}\, d\mathbf{s}.$$

(9) Let $\mathbf{F} = \langle y^2, x, z^2 \rangle$. Let S be the part of the paraboloid $z = x^2 + y^2$, below the plane $z = 1$, with the upward pointing normal. Verify Stokes' Theorem in this case by directly evaluating both integrals.

(10) Let E be the solid cylinder $x^2 + y^2 \leq 1$ with $0 \leq z \leq 3$, and let $\mathbf{F} = \langle x, y, -z \rangle$. Verify the divergence theorem by directly evaluating both integrals.