Soldis

MTH 231 College of Staten Island, CUNY Spring 2011

FINAL EXAM B

NAME:

Last 4 digits of your SS # :

ANSWER ALL QUESTIONS IN THE SPACE PROVIDED.

Please present clear solutions and fully explain your reasoning in complete sentences. Answers submitted without justification will not receive full credit. Only calculator is permitted.

GOOD LUCK!

1. Find the following limits (4 points each). You may use L'Hopital's rule where applicable.

(a) $\lim_{x \to 3} \frac{x-3}{x^2-9} =$

(b) $\lim_{x \to 0} \frac{\sin(3x)}{x} =$

(c) $\lim_{x \to \infty} \frac{x^2+5x-7}{5x^2+100x+3} =$

(d) $\lim_{x \to +\infty} \frac{x^2+5}{e^{2x}} =$

2. (4 points) Consider the following function

$$g(x) = \begin{cases} 5\cos x - 3 & x \leq 0 \\ 4x + b & x > 0 \end{cases}$$

Select b such that $g(x)$ should be continuous on the whole real line.

3. (4 points each) Find the first derivative for each of the following functions:

(a) $f(x) = x^4 + 4\,\mathrm{e}^x + \tan(x) + 3\pi + \mathrm{e}^4$

$f'(x) =$

(b) $f(x) = \sqrt{x} + \dfrac{x}{x^2 + 3}$

$f'(x) =$

(c) $f(x) = \ln(\sin x)$

$f'(x) =$

(d) $f(x) = x\,(\ln(x) - 2)$

$f'(x) =$

4. (6 points) Consider the curve described by the following equation $(x^2+x+1)y^2 = 3$ The point $(1,1)$ is on this curve. Use implicit differentiation to find the slope of the tangent line to the curve at this point. Write down the equation of the tangent line at the point $(1,1)$.

5.(15 points) Consider the following function:

$$g(x) = \left(\frac{1}{4}x^4 - x^2 - 2\right)$$

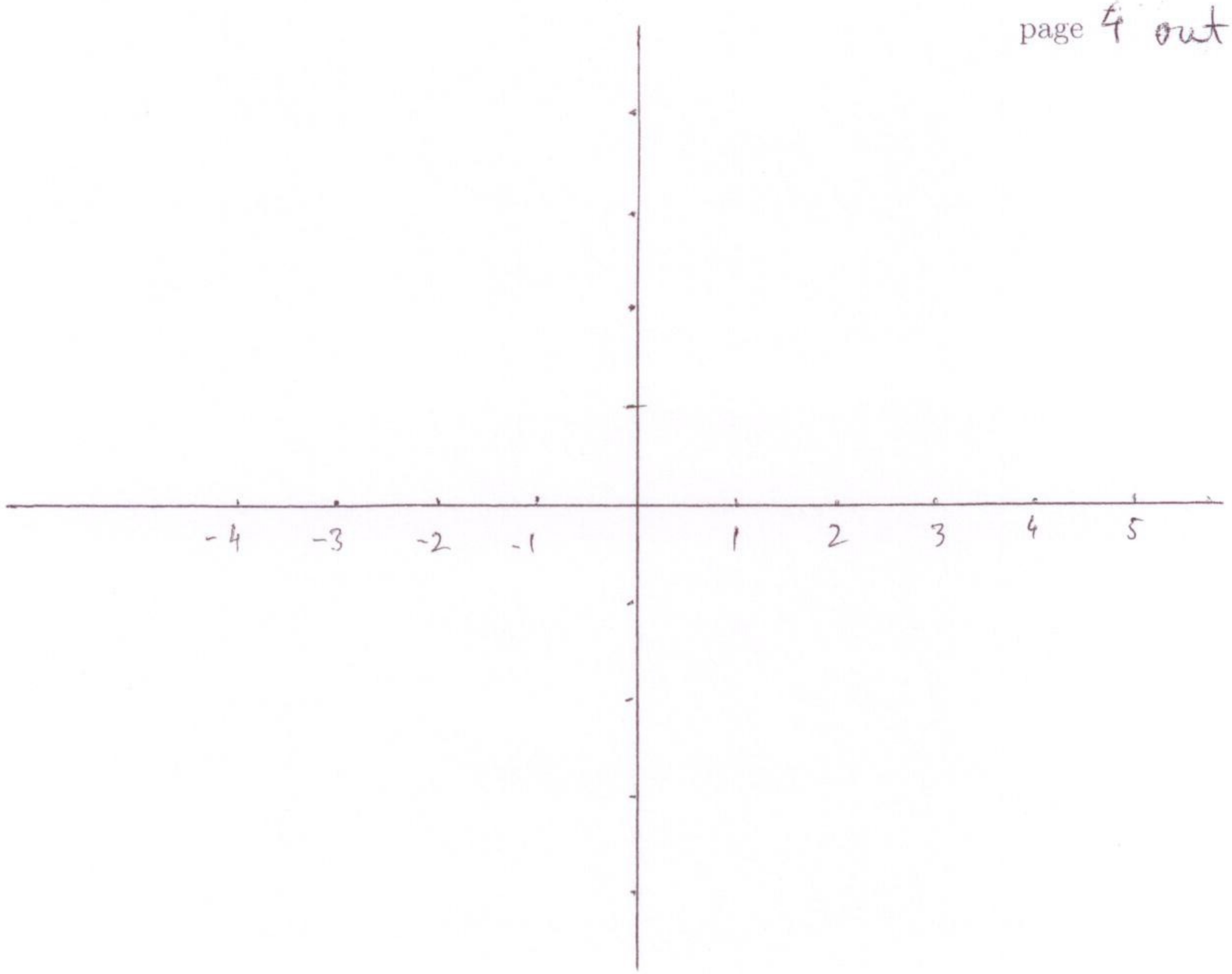
a) Calculate the derivative and find all critical points of the function.

b) Determine the interval(s) where g is increasing and those where g is decreasing. List them.

c) Use the first derivative test to find the coordinates of all relative maxima and minima.

d) Find, if they exist, the coordinates of all points of inflection and determine the intervals where g is concave up and those where g is concave down. List these intervals.

e) Sketch the curve as accurately as possible in the space provided on the next page. Label the points and indicate your findings in parts a-d.


-4
-3
-2
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1
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5

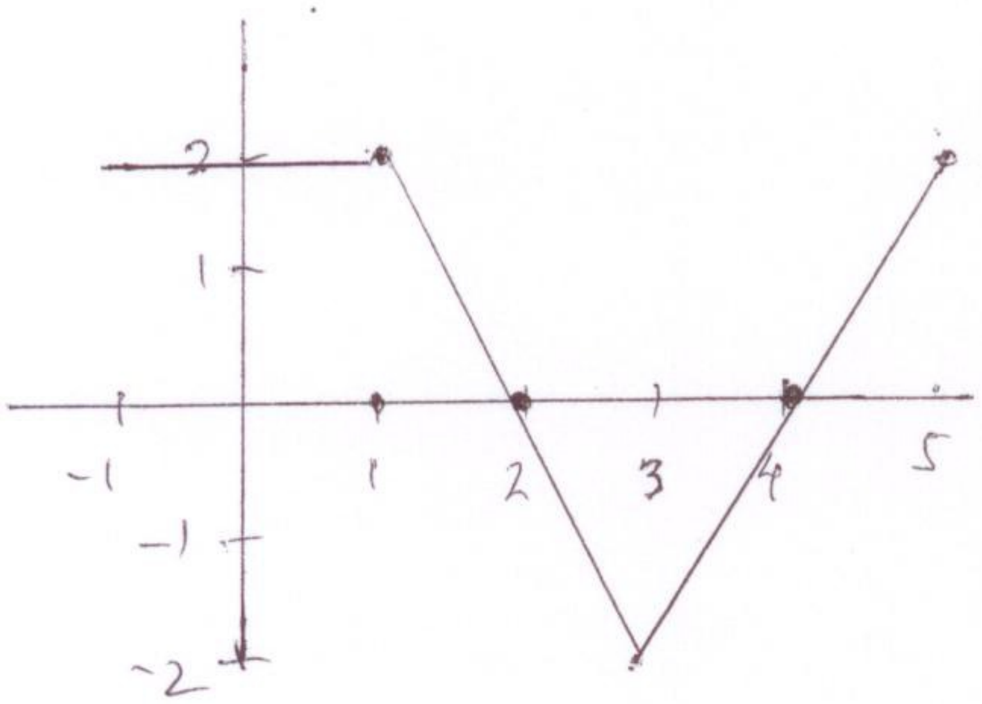
3 to two P signs

6. (6 points) Consider the function $f(x)$ defined by the graph below.

a. Sketch a graph of $f'(x)$ on the figure.

b. Calculate

$$\int_{-1}^{5} f(x)\,dx.$$



7. Find the following integrals (5 points each):

(a) $\int_1^2 \sqrt{x}+x^2+\frac{1}{x}\,dx =$

(b) $\int e^x + e^{-x} + 2\,dx =$

(c) $\int_0^{\pi/6} \sin(3x)\,dx =$

(d) $\int_0^2 e^{x^4} x^3\,dx =$

8./ **Do TWO out of the following four problems: MARK CLEARLY WHICH ONE YOU ARE DOING** (10 points each)

a./ Use $a(t) = -32$ feet per second as the acceleration due to gravity. A ball is thrown vertically upward **from the ground** with an initial velocity of 128 feet per second. How high will the ball go?

b./ Try to calculate the following integral in **two** different ways

$$\int_0^3 x^2\,dx$$

(i) by definite integration,
(ii) by using an approximating sum with n subintervals and taking the limit when $n \to \infty$.
Of course the two results has to be the same. You might need some of the following formulas:

$$\sum_{i=1}^{n} c = nc \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{[n(n+1)]^2}{4}$$

c./ Use linear approximation to calculate the approximate value of $\sqrt{103}$. When you are done, calculate $\sqrt{103}$ again by your calculator. Did you get similar answers?

d./ The squared distance between two points on the plane $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$.
Use calculus to determine which point on the graph of $y = \frac{1}{x}$, $x > 0$ is closest to the point $(0, 0)$.