MTH 231 College of Staten Island, CUNY Spring 2010

FINAL EXAM

NAME:

Last 4 digits of your SS # :

ANSWER ALL QUESTIONS IN THE SPACE PROVIDED.

Please present clear solutions and fully explain your reasoning in complete sentences. Answers submitted without justification will not receive full credit. Only calculator is permitted.

GOOD LUCK!

1. Find the following limits (4 points each). You may use L'Hopital's rule where applicable.

(a) $\lim_{x \to 3} \frac{x+3}{x^2-9} =$

(b) $\lim_{x \to 0} \frac{\sin(4x)}{x} =$

(c) $\lim_{x \to 0} \frac{x}{\sqrt{x+9}-3} =$

(d) $\lim_{x \to +\infty} \frac{-3x^4+2x^2-10}{5x^4-100x+3} =$

2. (4 points) Consider the following function

$$g(x) = \begin{cases} 5 - 3\cos x & x \leq 0 \\ 7x + b & x > 0 \end{cases}$$

Select b such that $g(x)$ should be continuous on the whole real line.

3. (4 points each) Find the first derivative for each of the following functions:

(a) $f(x) = x^3 + 3^x + e^x$

$f'(x) =$

(b) $f(x) = \dfrac{\sin x}{x^2 + 1}$

$f'(x) =$

(c) $f(x) = \sin(\sqrt{x})$

$f'(x) =$

(d) $f(x) = x^2 \ln(x)$

$f'(x) =$

4. (4 points) Consider the curve described by the following equation $x^3 + y^2 + y^4 = 10$ The point $(2, 1)$ is on this curve. Use implicit differentiation to find the tangent line to the curve at this point.

5.(15 points) Consider the following function:

$$g(x) = x\,(x-4)^3$$

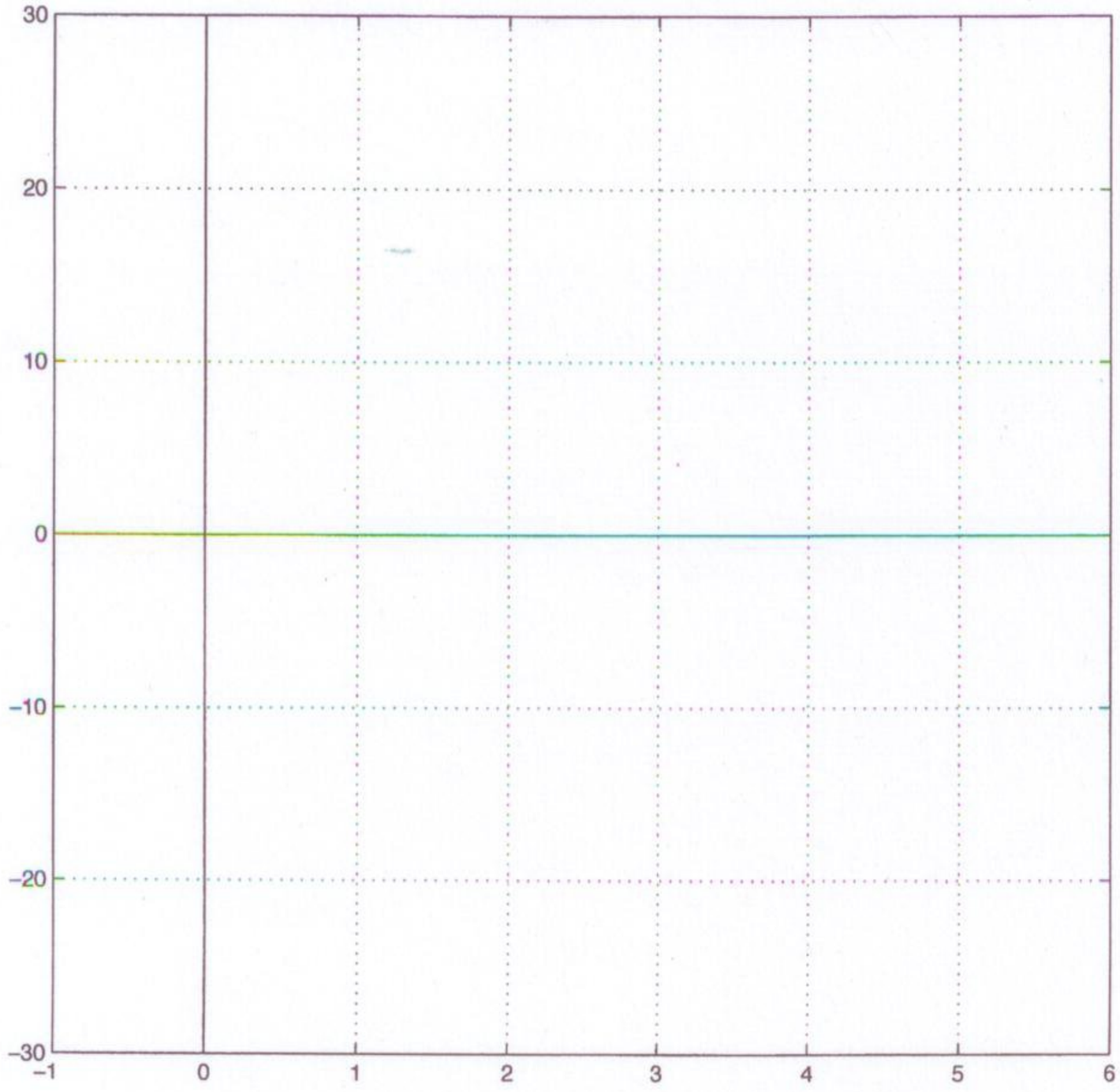
a) Calculate the derivative and find all critical points of the function.

b) Determine the interval(s) where g is increasing and those where g is decreasing.

c) Use the first derivative test to find the coordinates of all relative maxima and minima.

d) Find, if they exist, the coordinates of all points of inflection and determine the intervals where g is concave up and those where g is concave down.

e) Sketch the curve as accurately as possible in the space provided on the next page. Label the points and regions found in parts a-d.


30
20
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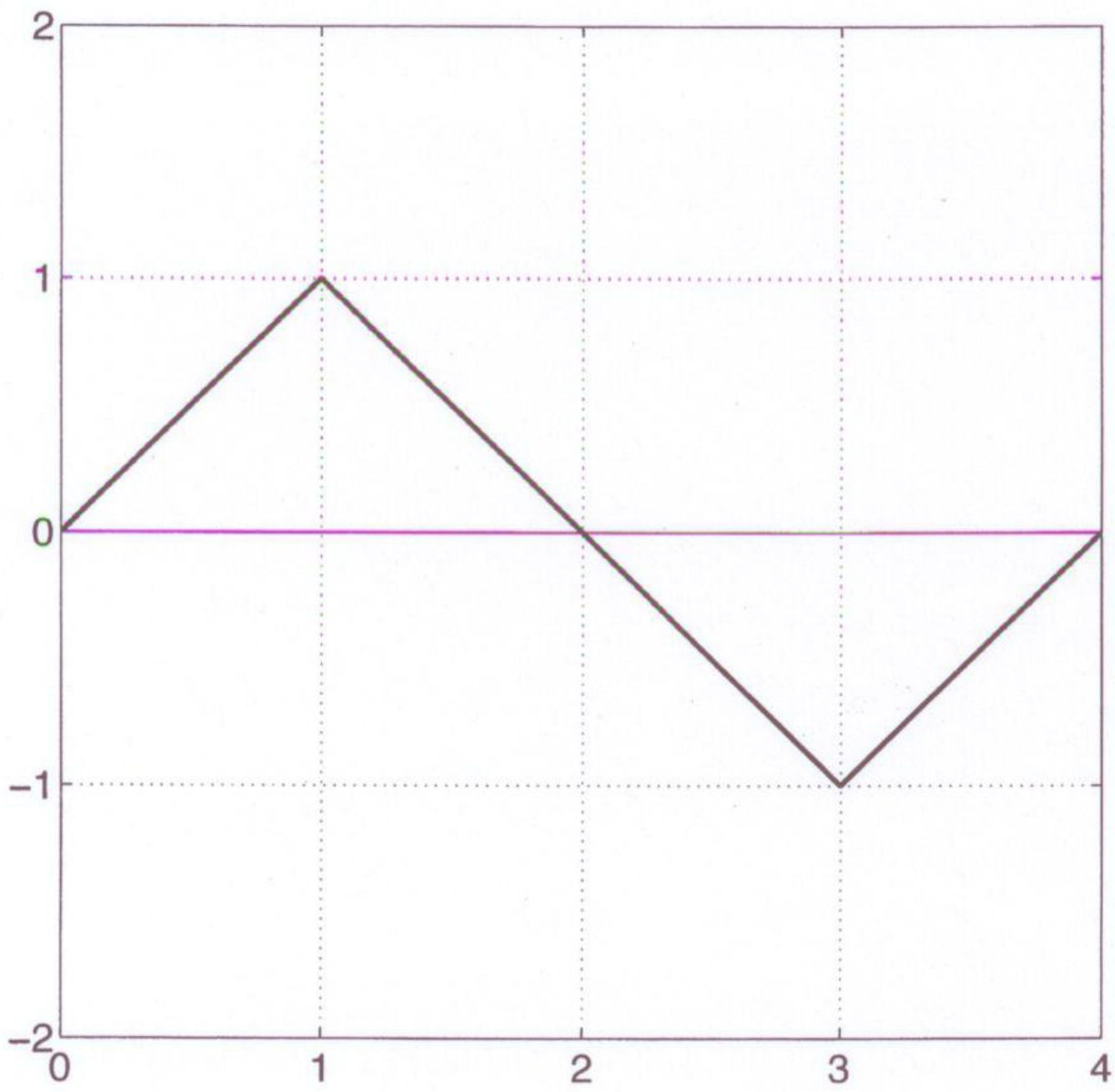
6

6. (5 points) Consider the function $f(x)$ defined by the graph below.



a. Sketch a graph of $f'(x)$ on the figure.

b. Calculate $\int_0^1 f(x)dx$.

c. What is the average value of $f(x)$ on the interval $[0, 2]$?

7. Find the following integrals (5 points each):

(a) $\int_1^3 x^2+\frac{1}{x}+\sqrt{x}\,dx =$

(b) $\int \frac{e^{2x}+3e^x+1}{e^x}\,dx =$

(c) $\int_0^{\pi/4} \sin(2x)\,dx =$

(d) $\int_0^1 e^{x^3}x^2\,dx =$

8./ Do TWO out of the following four problems: MARK CLEARLY WHICH ONE YOU ARE DOING (10 points each)

a./ Alex and Brenda are in motorboats located at the center of a lake. At time $t = 0$, Alex begins traveling south at a speed of 18 mph. At the same time, Brenda takes off heading east at a speed of 24 mph.

i. How far have Alex and Brenda each traveled after 1/6 of an hour?

ii. At what rate is the distance between Alex and Brenda increasing 1/6 of an hour after they start?

b./ Try to calculate the following integral in **two** different ways

$$\int_0^4 x^3\,dx$$

(i) by definite integration,
(ii) by using an approximating sum with n subintervals and taking the limit when $n \to \infty$. Of course the two results has to be the same. You might need some of the following formulas:

$$\sum_{i=1}^{n} c = nc \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{[n(n+1)]^2}{4}$$

c./ Use linear approximation to calculate the approximate value of $\sqrt{102}$. When you are done, calculate $\sqrt{102}$ again by your calculator. Did you get similar answers?

d./ A landscaper wishes to enclose a rectangular garden with a brick wall costing \$30/ft on one side and a metal fence costing \$10/ft on the other three sides. If the area enclosed is 1,000 ft^2, find the dimensions of the garden which minimize the cost.