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CSI, Dept of Math, Sp 2009

Calculus I (Math 231), Final Exam

Name: _____

Please check that you have **12 pages**, including this one. There are 100 points total.

There are 3 parts. **Do all problems in Part A** (Problems 1–6).
Do any ONE problem from Part B (Problems 7, 8).
Do any TWO problems from Part C (problems 9–11).

Note: You must label the problems you want graded in Parts B and C, otherwise the first problem(s) done will be graded. No extra credit for doing all problems—sorry!

Follow the policies of your instructor on the use of calculators and notes.

Cell-phones must be turned off and stowed.
Sharing of erasers, etc is not allowed.
Scratch paper must be obtained from the instructor and returned with the exam.

For full credit, **you must show work and/or give reasons for all answers.**

You are expected to do your own work and not discuss the test with anyone except the instructor. Copying or collaboration is grounds for a grade of zero. There may be multiple versions of this test.

Form 1

Part A: do all of problems 1–6

1. (*12 pts*) Find the following derivatives. Please do *not* simplify answers. (4 pts each)

a. Find the derivative $Q'(t)$, where $Q(t) = \dfrac{t^4 - 1}{\sqrt{t+5}}$.

b. Find the derivative $\dfrac{dy}{dx}$, where $y = e^{5x}(\sin x)^2$.

c. Find the *second* derivative $g''(x)$, where $g(x) = \ln(3x^2)$. *Suggestion*: log properties.

2. (*15 pts*) Find the following integrals. (5 pts each.)

a. $\displaystyle\int \frac{1 - 10x^3 + x^7}{x^3}\, dx$

b. $\displaystyle\int 4\cos^2\theta \, \sin\theta \, d\theta$

c. Compute the definite integral $\displaystyle\int_0^{\ln 5} 2e^{-t}\, dt$

3. (*10 pts*) *Note.* The possible answers for limits are a number, $+\infty$, $-\infty$ or "does not exist" (DNE).

a. (4 pts) Find $\lim_{x \to 4} \dfrac{x^2 - 3x - 4}{x - 4}$. Justify your answer.

b. (2 pts) Does $\dfrac{x^2 - 3x - 4}{x - 4}$ have a vertical asymptote at $x = 4$?
Explain, using your answer to Part (a).

c. (4 pts) Let $g(x) = \dfrac{x^2 - 25}{10x - 3x^2}$.
Find equations for all horizontal aymptotes, if any (using variables x and/or y). Justify your answer using limits.

4. (*10 pts*) Let $H(t) = \dfrac{3}{t-5}$

a. (4 pts) Sketch the graph of $H(t)$, showing any asymptotes.

b. (2 pts) On your graph, sketch and label the tangent line at the point where $t = 2$.

c. (4 pts) Find the slope of the tangent line at $t = 2$, using derivative rules. Show the calculation step by step.

5. (*13 pts*) Let $p(x) = 3x^4 - 6x^3 - 2$, so $p'(x) = 12x^3 - 18x^2$ and $p''(x) = 36x^2 - 36x$.

a. (5 pts) Give the intervals where $p(x)$ is increasing. _____

Give the intervals where $p(x)$ is decreasing. _____

Give the x values of all relative extrema, or write "none".

Maxima _____ Minima _____

Justify your answers using the appropriate derivative, showing all calculations.

Problem 5 continued: $p(x) = 3x^4 - 6x^3 - 2$.

b. (5 pts) Give the intervals where $p(x)$ is concave up. _____

Give the intervals where $p(x)$ is concave down. _____

Give the x values of all inflection points, or write "none". _____

Justify your answers using the appropriate derivative, showing all calculations.

c. (3 pts) Sketch a graph of $p(x)$, showing the concavity clearly, and label each maximum, minimum, and inflection point.

6. (*10 pts*) A region is bounded by the graph of $y = 9 - x^2$, the lines $x = 0$ and $x = 2$, and the x-axis.

a. (3 pts) Sketch and label the boundaries and shade in the region.

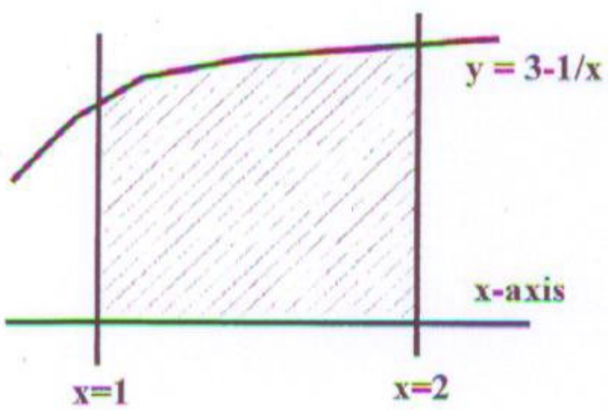
b. (7 pts) Compute the area of this region by writing down the appropriate integral and evaluating it.

Part B: do ONE of problems 7 or 8. (10 pts each.)
Circle the problem you want graded: 7 8

7. *(10 pts)* Let $f(x) = 3x^2 + 5$. Find the derivative $f'(x)$ using the *definition* (by computing a limit). Show all steps, being careful to use limit notation correctly.

8. (*10 pts*) A region is bounded by the graph of $h(x) = 3 - \frac{1}{x}$, the lines $x = 1$, $x = 2$ and the x-axis. (See sketch.)

a. (7 pts) Estimate the area of the region using a Riemann Sum with 4 rectangles of equal width. Use the right endpoint of each interval to find the rectangle height. Sketch the rectangles and show all steps of the calculation. (If no calculator—leave your answer as a sum, but simplify.)



b. (3 pts) Suppose the *left* endpoints are used instead of the right endpoints. Will the resulting Riemann Sum be larger, smaller, or the same as in Part (a)? Justify your answer. *Hint*: you do not need to do any calculations.

Part C: do any TWO of problems 9–11. (10 pts each.)
Circle the problems you want graded: 9 10 11

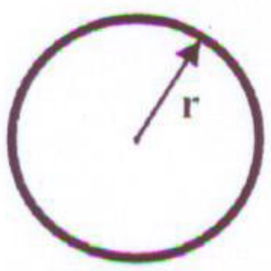
9. (*10 pts*) (Optimization.) A builder is designing a large rectangular window (see picture). The total length of the frame will be 60 feet: the total length is the perimeter of the outer rectangle plus the length of the two horizontal pieces.

Determine the dimensions (length and width) that maximize the total area of the window. Also find the maximum area. Include units.

Explain your reasoning step by step, and show all calculations.

10. (*10 pts*) Oil is leaking from from a boat creating a circular oil spill. The area A of the circle is increasing at a rate of 500 square meters per hour (m^2/hr). *Reminder*: area of a circle is πr^2, where r is the radius.

a. (5 pts) Find a formula for the rate at which the radius r is increasing, in terms of r, A, and/or dA/dt. Show your calculation step by step.



oil spill

b. (2 pts) Find the value of this rate when the radius reaches 50 meters. Include units.

c. (3 pts) As the radius continues to grow, is this rate constant, increasing, or decreasing? Explain your answer.

11. (*10 pts*) A curve is described by the equation $x^3 + y^2 = 2xy + 1$.

a. (6 pts) Using implicit differentiation, find a formula for the derivative $\frac{dy}{dx}$ in terms of x and/or y. Show your calculation step by step.

b. (2 pts) Find the slope of the tangent line to the curve at the point (1, 2).

c. (2 pts) Give an *equation* for this tangent line.