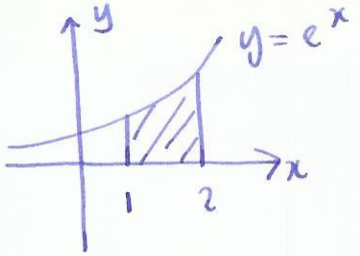
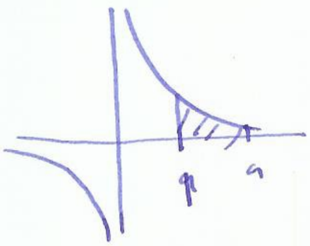


$$\int_1^2 e^x\,dx = \left[e^x\right]_1^2 = e^2 - e^1$$



$$\int_1^a \frac{1}{x}\,dx = \left[\ln|x|\right]_1^a = \ln(a) - \ln(1) = \ln(a).$$

observations

① choice of anti-derivative doesn't matter: let $F(x)$ and $F(x)+c$ be anti-derivatives for $f(x)$. Then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$
$$= F(b)+c - \big(F(a)+c\big) = F(b) - F(a).$$

② $\int_a^t f(x)\,dx$ is a function of t! x is a dummy variable.

i.e. $\int_a^t f(x)\,dx = \int_a^t f(y)\,dy$, if you want function of x: $\int_0^x f(t)\,dt$

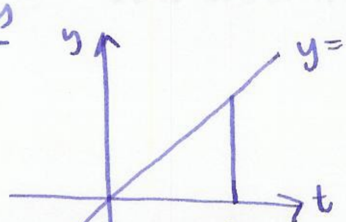
§ 5.4 Fundamental theorem of calculus II

Thm (FTC②) Let $f(x)$ be cts on $[a,b]$, then

$A(x) = \int_a^x f(t)\,dt$ is an anti-derivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx}$

i.e. $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. Furthermore $A(a) = 0$

Examples



$$\int_0^x t\,dt = \left[\tfrac{1}{2}t^2\right]_0^x = \tfrac{1}{2}x^2 - 0 = \tfrac{1}{2}x^2$$

$\int_0^x e^{-t^2}\,dt$ ← a function with anti-derivative e^{-x^2}.

<u>Example</u> what about $\int_0^{x^2} \sin(t)\,dt$ ← this is a function of a function!

set $A(x) = \int_0^x \sin(t)\,dt$, then if $G(x) = \int_0^{x^2} \sin(t)\,dt$

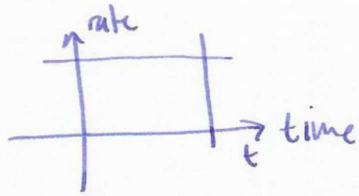
$G(x) = A(x^2)$ ← use chain rule to differentiate

$\frac{d}{dx}\left(A(x^2)\right) = A'(x^2).\,2x = \sin(x^2).\,2x$.

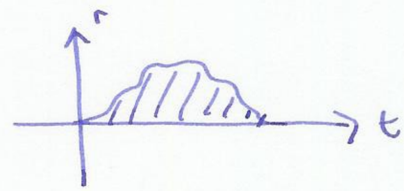
§5.5 Net change / applications of integrals

<u>Example</u> you pour water into a bucket at rate $r(t)$. How much water is in the bucket?

constant rate:



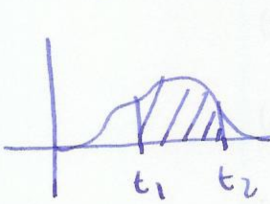
amount = rate × time



varying rate: amount = area under the curve

$$= \int_0^t r(x)\,dx.$$

<u>Q</u>: by how much did the amount of water change between t_1 and t_2?



$$\text{net change} = \int_{t_1}^{t_2} r(x)\,dx$$

<u>Example</u> water flows into the reservoir at rate $3000 + 5t$ gallons / hour. how much water goes in during the 4th hour?

$$\int_3^4 3000 + 5t\,dt = \left[3000t + \frac{5}{2}t^2\right]_3^4 = 3000 + \frac{5}{2}(16-9) = 3000 + \frac{35}{2} \text{ gallons}.$$