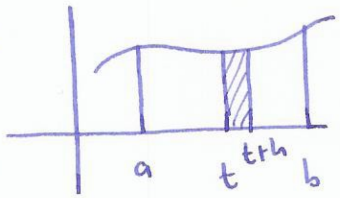
§5.3 Fundamental theorem of calculus

Thm (FTC 1) Suppose $f(x)$ is continuous on $[a,b]$ and $F(x)$ is an anti-derivative for $f(x)$. Then $\int_a^b f(x)\,dx = F(b) - F(a)$.

intuition



consider $\int_a^t f(x)\,dx$

Q: what is rate of change wrt t?

adding $\frac{d}{dt}\int_a^t f(x)\,dx = \lim_{h\to 0} \dfrac{\int_a^{t+h} f(x)\,dx - \int_a^t f(x)\,dx}{h} = \lim_{h\to 0} \dfrac{\int_t^{t+h} f(x)\,dx}{h}$

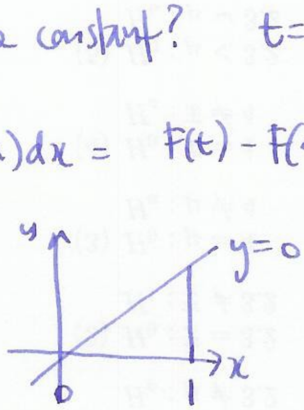
$\underset{\text{approx}}{\approx} \dfrac{\text{rectangle of area } f(t)\times h}{h} = f(t).$

i.e. $\int_a^t f(x)\,dx$ is an anti-derivative for $f(x)$, so $\int_a^t f(x)\,dx = F(t) + c$

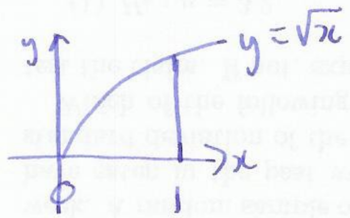
what is the constant? $t=a$: $\int_a^a f(x)\,dx = 0 = F(a) + c = 0$ so $c = -F(a)$

so $\int_a^t f(x)\,dx = F(t) - F(a)$.

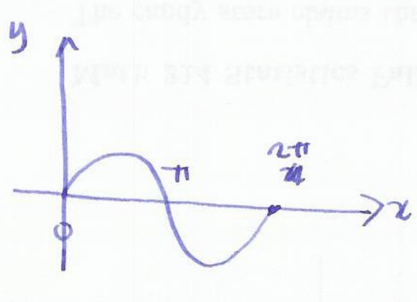
Examples



$\int_0^1 x\,dx = \left[\frac{1}{2}x^2\right]_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$



$\int_0^1 x^{1/2}\,dx = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3} - 0 = \frac{2}{3}$



$\int_0^\pi \sin(x)\,dx = \left[-\cos(x)\right]_0^\pi = -\cos(\pi) + \cos(0)$

$= -(-1) + 1 = 2$