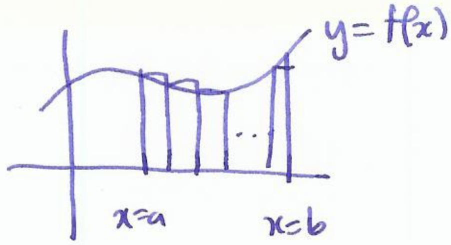
Notation



N rectangles of equal width

then $\Delta x = \dfrac{b-a}{N}$

then:

left endpoint rectangles $L_N = \displaystyle\sum_{j=0}^{N-1} f(a + j\Delta x)\,\Delta x$

right endpoint rectangles $R_N = \displaystyle\sum_{j=1}^{N} f(a + j\Delta x)\,\Delta x$

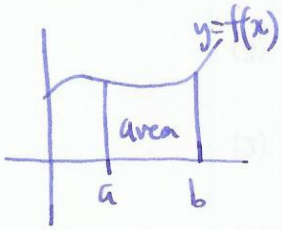
midpoint rectangles $M_N = \displaystyle\sum_{j=1}^{N} f\left(a + (j-\tfrac{1}{2})\Delta x\right)\Delta x$

useful fact:

Thm If $f(x)$ is continuous on $[a,b]$ then all three approximations give the same limit as $N \to \infty$, which is the area under the curve.

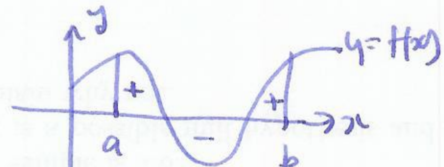
$$\text{area} = \lim_{N\to\infty} L_N = \lim_{N\to\infty} R_N = \lim_{N\to\infty} M_N$$

§5.2 Definite integral

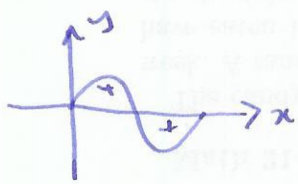


$\displaystyle\int_a^b f(x)\,dx$ = area under the curve $y=f(x)$ between $x=a$ and $x=b$.

note: signed area:



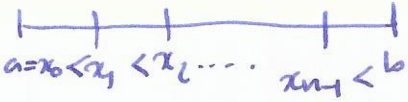
so



$\displaystyle\int_0^{2\pi} \sin(x)\,dx = 0$

Formal definition: Riemann Sum $R(f, P, C)$

$P =$ partition of $[a,b]$



widths $\Delta x_i = x_i - x_{i-1}$

$C =$ choice of points $c_i \in [x_{i-1}, x_i]$

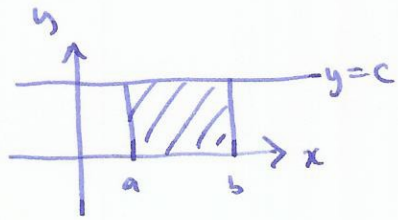
$$R(f,P,C) = \sum_{i=1}^{n} f(c_i)\Delta x_i \qquad \|P\| = \max \Delta x_i$$

<u>Def^n</u> $$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} R(f,P,C) = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

when this limit exists, we say f is <u>integrable</u> over $[a,b]$.

<u>useful properties</u>

$$\int_a^b c\,dx = c(b-a)$$



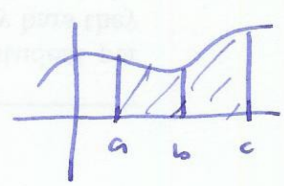
$$\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

reversing limits: $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

0-length interval: $\displaystyle\int_a^a f(x)\,dx = 0$

adjacent intervals: $\displaystyle\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



comparisons: if $f(x) \le g(x)$ then $\displaystyle\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$