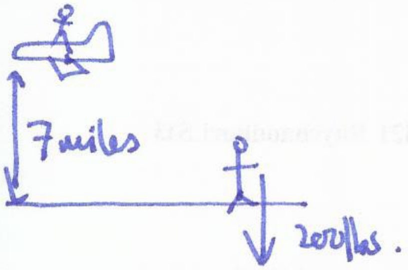
Example weight loss



$$W(x) = \frac{wR^2}{x^2}$$

$R = 3960$ miles (radius of earth)

$x =$ distance from center of earth

$w =$ weight at ground ($R = 3960$).

$$w'(x) = -\frac{2wR^2}{x^3}$$

$$w(3967) \approx w(3960) + w'(3960)\Delta x$$

$$\approx 200 + -2 \cdot 200 \cdot \frac{(3960)^2}{(3960)^3} \cdot 7 \approx 200 - 0.7 .$$

Example pizza size: you make an 18" pizza, if your diameter is accurate to ± 0.4in how much pizza could you gain or lose?

$A = \pi r^2 \qquad 2r = D \qquad A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$

$$A'(D) = \frac{2\pi D}{4} = \frac{\pi D}{2}$$

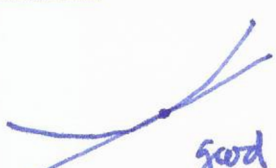
$$\Delta A \approx A'(18) \cdot \Delta D = \frac{1}{2}\pi\, 18 \cdot 0.4 \approx 11 \text{ in}^2 .$$

Q: is this good or bad?

absolute error $= 11$

percentage error $= \left| \frac{\text{error}}{\text{actual value}} \right| \times 100 = \left(\frac{11}{\pi 18^2/4} \right) \times 100 .$

observation: when is the linear approximation a good approximation?

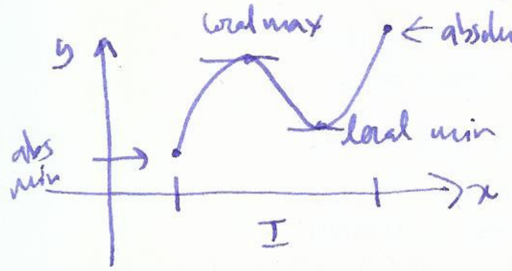


good. $f''(x)$ small

bad $f''(x)$ large.

§4.2 Extreme Values (max/min)

Suppose $f(x)$ is defined on an interval I



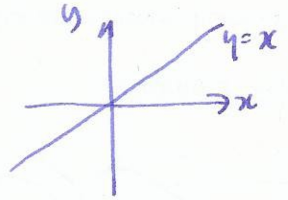
Def$^{\underline{n}}$ · $f(a)$ is the absolute max if $f(a) \geq f(x)$ for all $x \in I$

· $f(a)$ is the absolute min if $f(a) \leq f(x)$ for all $x \in I$.

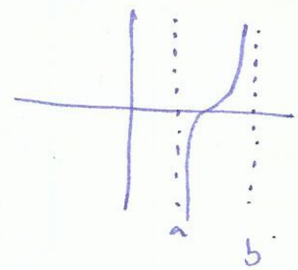
note: for absolute max or min, usually want value of function, not x-value.

warning: some functions don't have any max or min

Example: $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x$

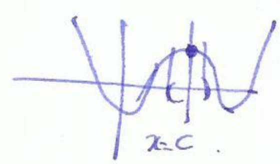


open intervals (a,b)



Th$^{\underline{m}}$ If $f(x)$ is continuous on a closed and bounded interval then $f(x)$ has both an absolute max and an absolute min.

Def$^{\underline{n}}$ $f(x)$ has a local max at $x=c$ if $f(c)$ is the maximum value of f for some small interval containing c. Similarly for local min.



Def$^{\underline{n}}$ we say c is a critical point if $f'(c)=0$

Th$^{\underline{m}}$ If c is a local max or min, then $f'(c)=0$.

Warning $f'(c)=0 \not\Rightarrow$ local max or min.

Example $y=x^3$

$f'(x) = 3x^2 = 0$ when $x=0$

but $x=0$ not local max or min.

How to find absolute max/min of functions on closed intervals $[a,b]$

① find critical points, and evaluate function there

② check end points.

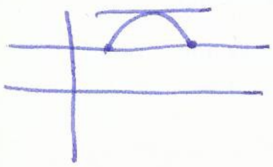
<u>Example</u> find abs max/min of $2x^3-15x^2+24x+7$ on $[0,5]$

x^2-8 on $[1,4]$

$\sin x \cos x$ on $[0,\pi]$.

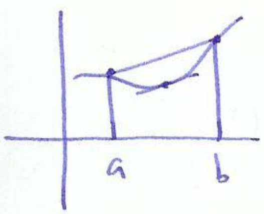
<u>Thm</u> (Rolle's Thm) Suppose $f(x)$ is ~~differentiable~~ continuous on $[a,b]$ and differentiable on (a,b). If $f(a)=f(b)$ then there is a $c\in[a,b]$ s.t. $f'(c)=0$.

<u>Proof</u>



- if there is a local max or min, then there is a c s.t. $f'(c)=0$
- if no local max/min $f(x)=\text{const} \Rightarrow f'(c)=0$ for all c.

§4.3 First derivative test



<u>Thm</u> (MVT)(Mean Value Theorem)

Suppose f is continuous on $[a,b]$ and differentiable on (a,b) then there is a $c\in(a,b)$ s.t. $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

i.e. there is a point where the slope is equal to the average rate of change.

<u>Corollary</u> If $f(x)$ is differentiable and $f'(x)=0$, then $f(x)=c$ constant

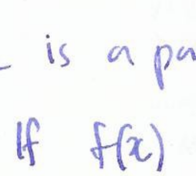
<u>Proof</u> spose there is a,b s.t. $f(a)\neq f(b)$. Then $\exists\, c\in(a,b)$ with $f'(c)=\dfrac{f(b)-f(a)}{b-a}\neq 0$ * $\square$

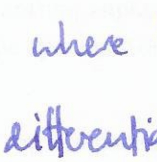
<u>Monotonicity</u> suppose f is differentiable on (a,b):

If $f'(x)>0$ for all $x\in(a,b)$ then f is increasing on (a,b)

$f'(x)<0$ decreasing

<u>Example</u> ① $f(x)=x^2$ $f'(x)=2x$





increasing on $(0,\infty)$

decreasing on $(-\infty,0)$

② $f(x)=x^2-2x-3$.

$f'(x)=2x-2$

$f'(x)>0$ when $2x-2>0$

$x>1$.